

CMSC 3540 I: The Interplay of Learning and Game Theory (Autumn 2022)

Prediction Markets (as a Forecasting Tool)

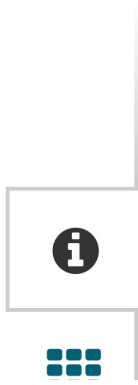
Instructor: Haifeng Xu



JOURNAL ARTICLE

Orange Juice and Weather




Richard Roll



The American Economic Review
Vol. 74, No. 5 (Dec., 1984),
pp. 861-880 (20 pages)

Futures of orange juice can be used to predict weather



| SAT | SUN | MON |
|---|---|---|
|  |  |  |
| Mostly Sunny | T-storms | Sunny |
| High: 76° Low: 52° | High: 72° Low: 37° | High: 55° Low: 32° |

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Events of Interest for Prediction

- Will there be a HW3 for this course?
- Will UChicago faculties win a Nobel Prize in 2023?
- Will bit coin price exceed \$25K tomorrow?
- Will Tesla's stock exceed \$250 by the end of this year?
- Will the number of iPhones sold in 2022 exceed 200 million?
- Will there be a cure for cancer by 2025?
- Will the world be peaceful in 2050?
- . . .

The Prediction Problem

- An uncertain event to be predicted
 - E.g., will Tesla stock exceed \$250 by Dec 2022?
- Dispersed information/evidence
 - E.g., Tesla employees, Tesla drivers, other EV company employees, government policy makers, etc.
- Goal: generate a prediction that is based on information aggregated from all sources
 - ML can also do prediction, but will see why markets have advantages

Bet \approx Credible Opinion

Q: will P vs NP problem be solved by the end of 20'th century?

P vs NP would be solved by the end of the 20th century, if not sooner.
Betted **one ounce of pure gold** with



Michael Sipser

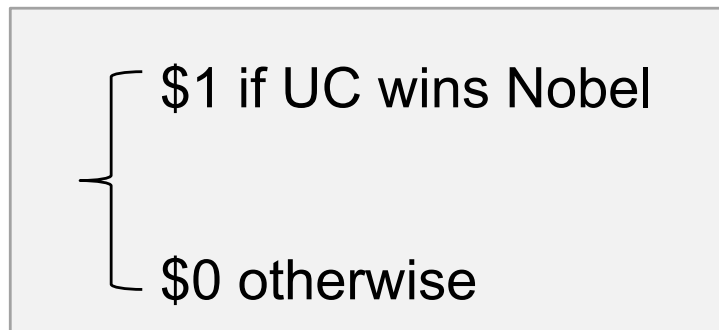


- Other examples: stock trading, gambling, . . .
- Betting intermediaries: Wall Street, Las Vegas, InTrade, . . .

Prediction Markets

A prediction market is a **financial market** that is designed for **event prediction** via information aggregation

- Payoffs of the traded financial **contract** are determined by outcomes of future events








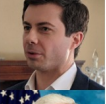

A (financial) contract

Price of a contract? $\$1 \times$ percentage of shares that bet on UC wining?

This is what we will be designing!

Prediction Markets: Examples

The screenshot shows the Predict It website interface. The top navigation bar includes 'Predict It', 'Markets', 'Support', 'Insights', and 'Leaderboard'. There are 'Login' and 'Sign Up' buttons, and a search icon. Below the navigation bar, there are category links: 'U.S. Elections', 'Trump Admin', 'Congress', 'Justice', and 'World'. The main heading is 'Who will win the 2020 U.S. presidential election?'. Below this is a table with columns for 'Contract', 'Latest Yes Price', 'Best Offer', and 'Best Offer'. The table lists candidates with their respective prices and 'Buy Yes'/'Buy No' buttons.


| Contract | Latest Yes Price | Best Offer | Best Offer |
|--|------------------------|------------|------------|
|  Donald Trump | 41¢ <small>NC</small> | 41¢ | 60¢ |
|  Elizabeth Warren | 31¢ <small>NC</small> | 32¢ | 69¢ |
|  Joe Biden | 13¢ <small>NC</small> | 13¢ | 88¢ |
|  Bernie Sanders | 11¢ <small>2¢↑</small> | 11¢ | 91¢ |
|  Andrew Yang | 8¢ <small>NC</small> | 9¢ | 92¢ |
|  Pete Buttigieg | 7¢ <small>NC</small> | 7¢ | 94¢ |
|  Mike Pence | 3¢ <small>NC</small> | 3¢ | 98¢ |

Prediction Markets: Examples

Predict It
Markets
Insights
Leaderboards
Support
Login
Sign Up
🔍

Biden Administration
U.S. Elections
Congress
State/Local
World

Will Andrew Cuomo resign before May 1?



Latest Price

Latest Yes Price

11¢ 2¢↓

Best Offer

11¢

Best Offer

90¢

Buy Yes

Buy No

The Rules

This market shall resolve to Yes in the event that Andrew Cuomo resigns from, and ceases to hold, the office of Governor of New York by the End Date listed below.

Read the Full Rules

Related Markets

Cuomo in office at the end of the year?

Yes: 52¢ 3¢↑

No: 48¢ 3¢↓

2.5M Shares Traded

New York Dem gubernatorial nominee?

Letitia James: 40¢ 2¢↓

Andrew Cuomo: 16¢ NC

190K Shares Traded

Primary (D)

Will Cuomo be impeached before 9/1?

Yes: 28¢ 1¢↑

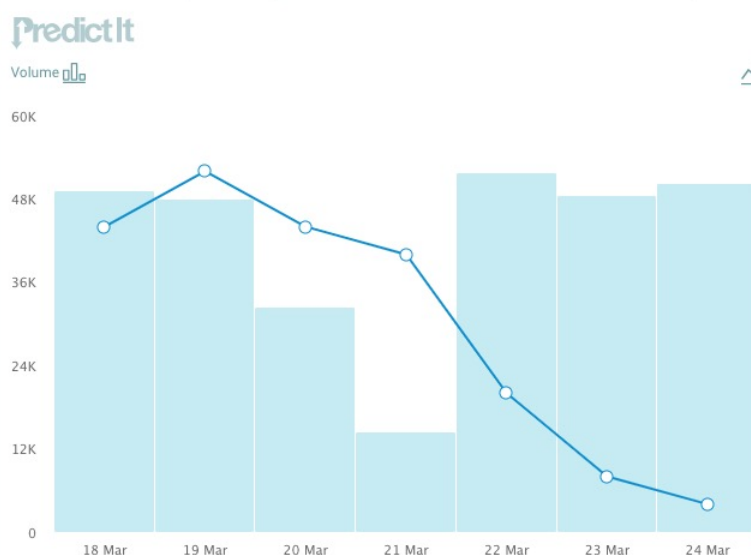
No: 72¢ 1¢↓

77,236 Shares Traded

Stats
24hr
7 Day
30 Day
90 Day
Candle
Line

Predict It

Volume 📊



| Date | Price |
|--------|-------|
| 18 Mar | 48¢ |
| 19 Mar | 52¢ |
| 20 Mar | 48¢ |
| 21 Mar | 45¢ |
| 22 Mar | 35¢ |
| 23 Mar | 25¢ |
| 24 Mar | 11¢ |

Price

9

Prediction Markets: Examples

← → ↻ replicationmarkets.com

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Reliable research replicates...
You can bet on it.

SCORE Markets Resolved
Per our 16-NOV-2021 announcement, SCORE replication markets had 121 resolutions: 79 regular & 30 covid replications, plus 12 meta-claims. We contacted [the 258 winners](#) to claim \$142,623 in prizes, and this has now closed. Thanks to our forecasters for hard work, and to DARPA SCORE for the generous funding.
~C. Twardy, PI

Announcing COVID-19 Preprint Market Winners
03-Feb-2022
The [resolutions for the prediction market](#) for the 400 preprints related to COVID-19 have been finalized, and prizes have been calculated.
Winners have been notified by email at the address registered with the username on the Replication Markets platform.

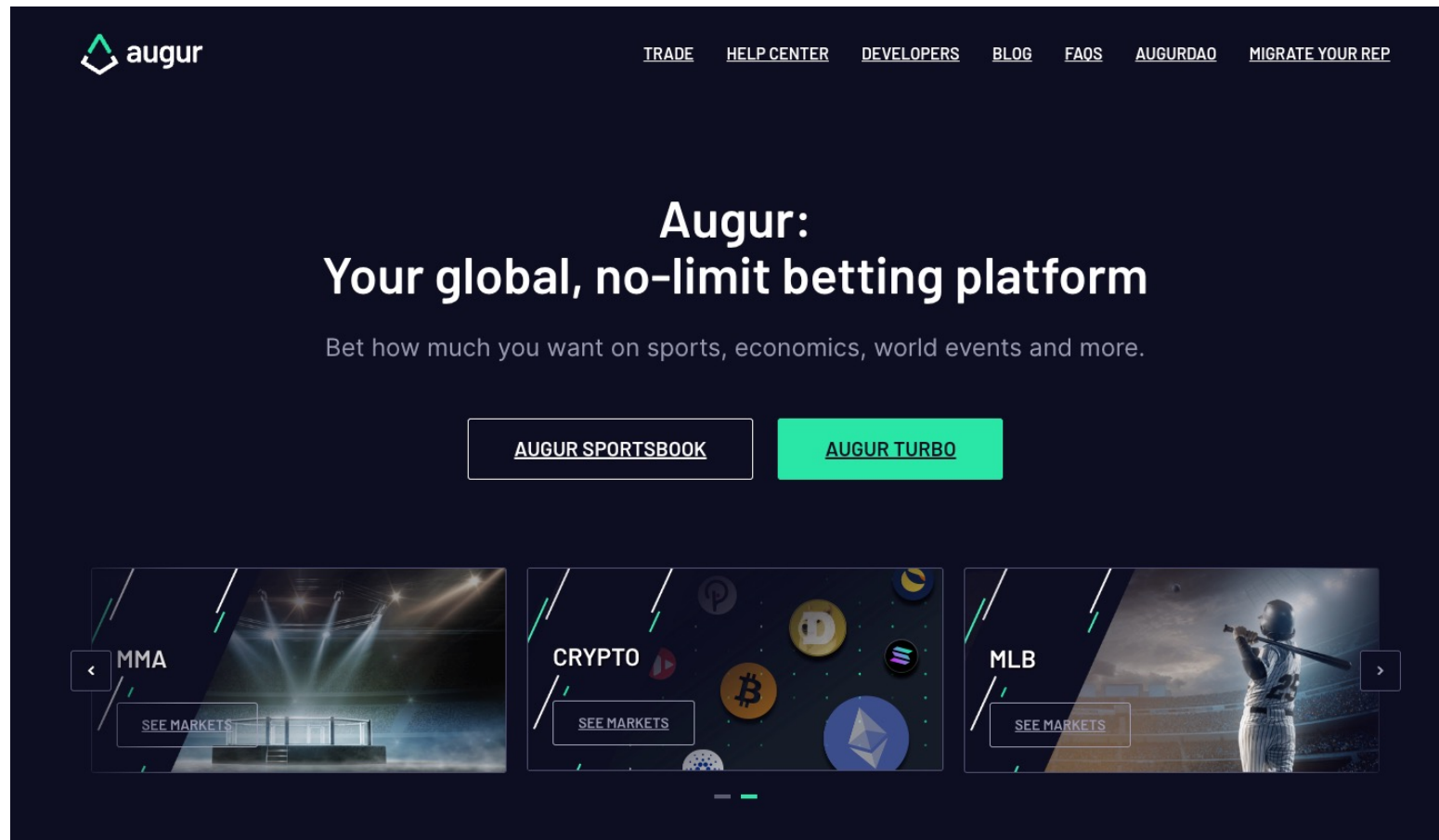
Other Projects
RM is done, but our research continues at the [Science Prediction Markets Project](#).
You may also like BITSS' [Social Science Prediction Platform](#).

[Go to SPMP](#)

[Go to RM's COVID-19 Site](#)

Replication Market

Prediction Markets: Examples



First decentralized prediction market (built on Ethereum blockchain)

Does It Work?

- Yes, evidence from real markets, lab experiments, and theory
 - I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995][Rietz 1998][Berg 2001][Pennock 2002]
 - HP market beats sales forecast 6/8 [Plott 2000]
 - Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC'03][Schmidt 2002]
 - Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC'01]
 - Theory: “rational expectations” [Grossman 1981][Lucas 1972]
 - More ...

Why Can Markets Aggregate Information?

➤ Price $\approx Prob(\text{event} | \text{all information})$

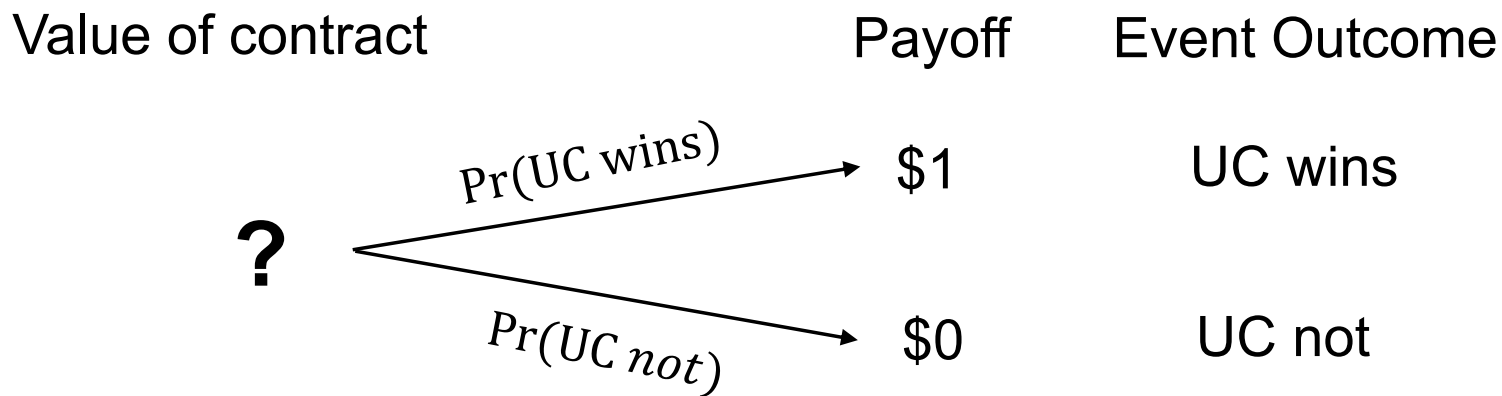
\$1 if UC wins Nobel, \$0 otherwise

| Value of contract | Payoff | Event Outcome |
|-------------------|--------|---------------|
| ? | \$1 | UC wins |
| | \$0 | UC not |

Why Can Markets Aggregate Information?

➤ Price $\approx \text{Prob}(\text{event} | \text{all information})$

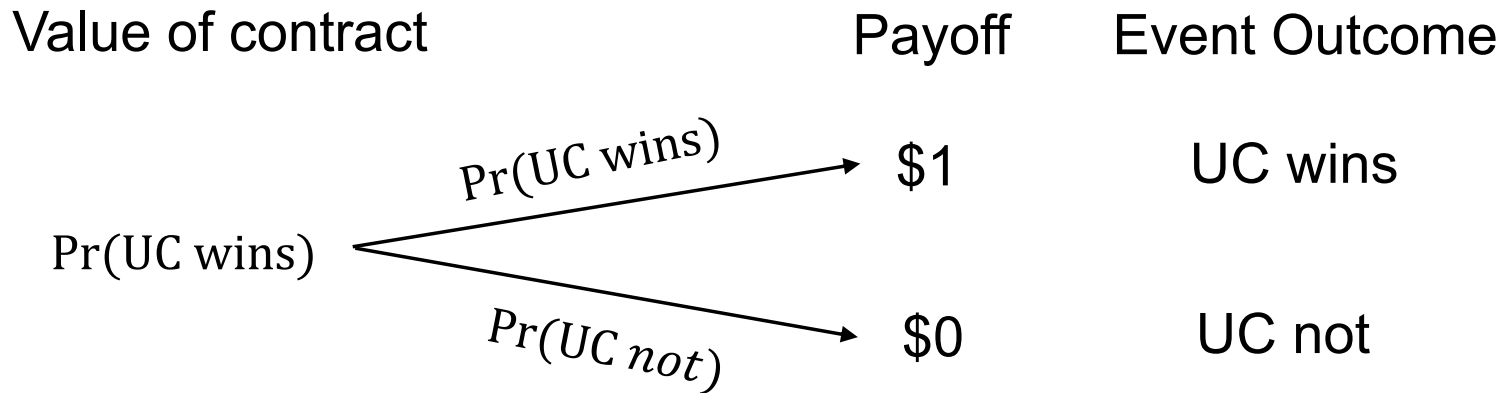
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Why Can Markets Aggregate Information?

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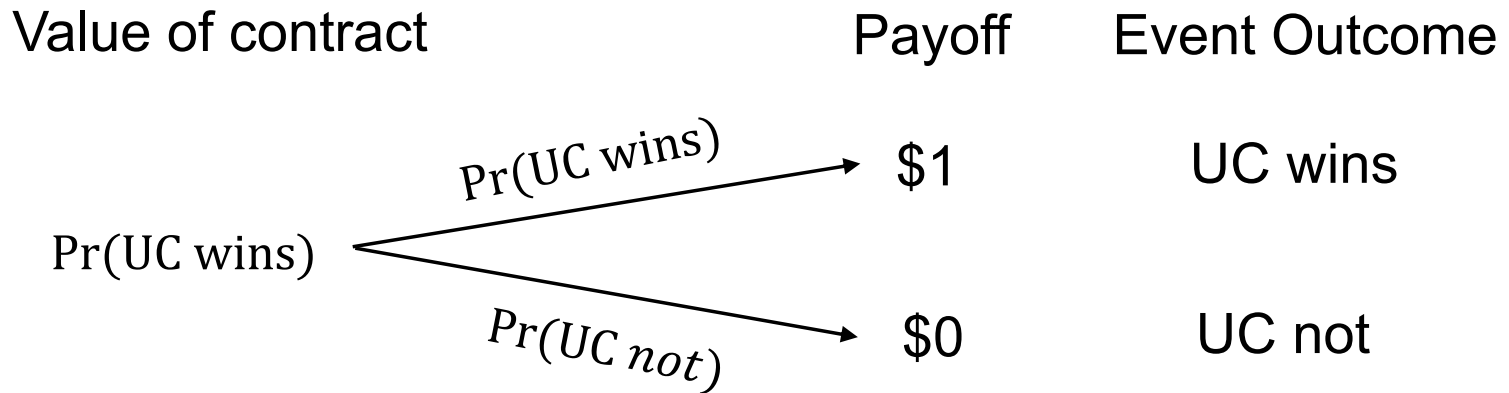


Value of contract \approx P(UC wins)

Why Can Markets Aggregate Information?

➤ Price \approx Prob(event| all information)

\$1 if UC wins Nobel, \$0 otherwise



Value of contract \approx P(UC wins) \approx Equilibrium price

Market Efficiency (a design goal)

Markets vs Other Prediction Approaches

Opinion Poll

- Sampling
- No incentive to be truthful
- Equally weighted information
- Hard to be real-time

Ask Experts

- Identifying experts can be hard
- Combining opinions is difficult

Prediction Markets

- Self-selection
- Monetary incentive and more
- Bet-weighted information
- Real-time
- Self-organizing

Other Prediction Approaches vs Markets

Machine Learning

- Historical data
- Assume past and future are related
- Hard to incorporate recent new information

Prediction Markets

- No need for data
- No assumption on past and future
- Immediately incorporate new information

Caveat: markets have their own problems too – manipulations, irrational traders, etc.

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets (PMs)
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Some Design Objectives of PMs

Liquidity: people can find counterparties to trade whenever they want

Bounded loss: total loss of the market institution is bounded

Market efficiency: Price reflects predicted probabilities.

Computational efficiency: The process of operating the market should be computationally manageable.

Continuous Double Auction (CDA) Market

\$1 if UC wins Nobel, \$0 otherwise

➤ Buyer orders

\$0.12

\$0.09

\$0.15

➤ Seller orders

\$0.30

\$0.17

\$0.13

Continuous Double Auction (CDA) Market

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\$0.17

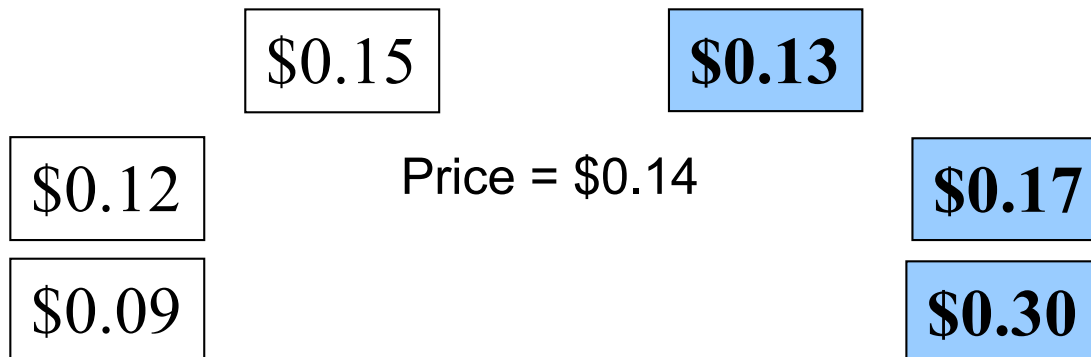
\$0.30

Continuous Double Auction (CDA) Market

\$1 if UC wins Nobel, \$0 otherwise

➤ Buyer orders

➤ Seller orders



What are Issues with CDA?

- Thin market problem
 - When there are not enough matches, trade may not happen.
- No trade theorem [[Milgrom & Stokey 1982](#)]
 - Why trade? These markets are zero-sum games (negative sum w/ transaction fees)
 - For all money earned, there is an equal (greater) amount lost; am I smarter than average?
 - Rational risk-neutral traders will *never* trade
 - But trade happens ...

An Alternative: Market Maker (MM)



- A market maker is the market institution who sets the prices and is willing to accept orders (buy or sell) at the price specified.
- Why? **Liquidity!**
- Market makers bear risk. Thus, we want mechanisms that can **bound the loss of market makers**.

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- An automated market marker (MM)
- Sell or buy back contracts $\boxed{\$1 \text{ iff } e_1}$ \dots $\boxed{\$1 \text{ iff } e_n}$
- Described by a “value function” ($q = (q_1, \dots, q_n) \in \mathbb{R}^n$ is sold #contracts)

$$V(q) = b \ln \sum_{j \in [n]} e^{q_j/b}$$

Parameter b
adjusts liquidity

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ An automated market marker (MM)

➤ Sell or buy back contracts

\$1 iff e_1

• • •

\$1 iff e_n

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$$V(q) = b \ln \sum_{j \in [n]} e^{q_j/b}$$

Parameter b
adjusts liquidity

➤ To buy $x \in \mathbb{R}^n$ amount, a buyer “pays”: $V(q + x) - V(q)$

- Negative x_i 's mean selling contracts to MM
- Negative payment means market maker pays the buyer
- Market starts with $V(0) = b \ln n$

➤ Price function

$$p_i(q) = \lim_{\delta \rightarrow 0} \frac{V(q + \delta e_i) - V(q)}{\delta}$$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ An automated market marker (MM)

➤ Sell or buy back contracts

$$\boxed{\$1 \text{ iff } e_1} \quad \cdot \cdot \cdot \quad \boxed{\$1 \text{ iff } e_n}$$

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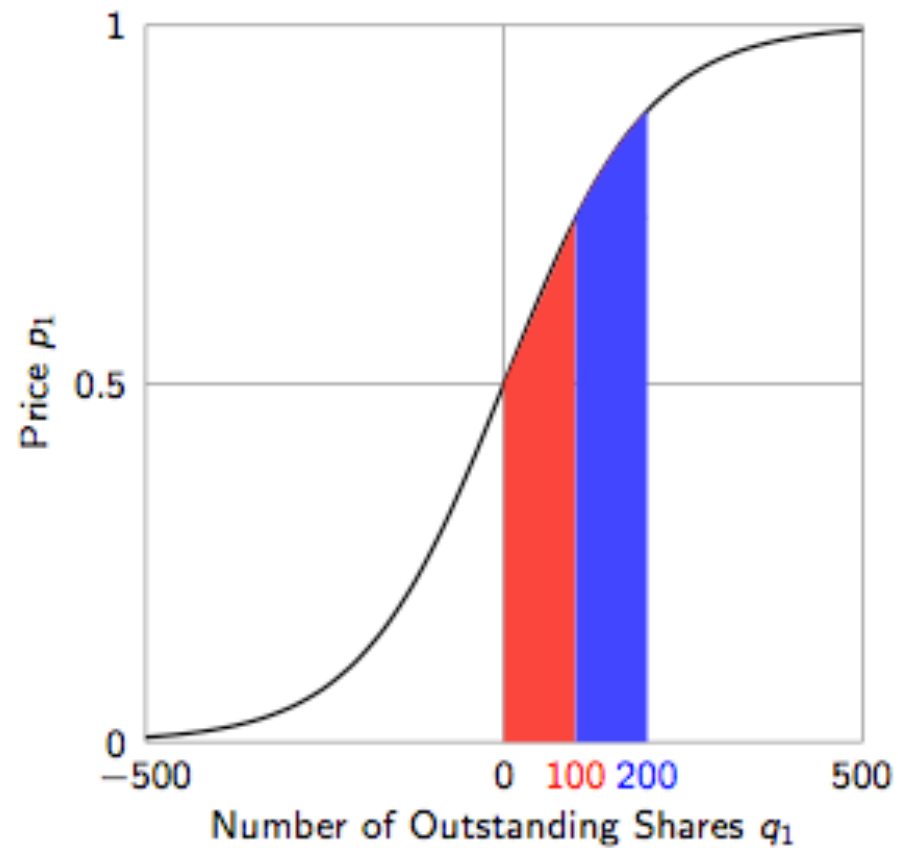
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➤ Price function

$$p_i(q) = \lim_{\delta \rightarrow 0} \frac{V(q + \delta e_i) - V(q)}{\delta} = \frac{\partial V(q)}{\partial q_i} = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

Price Curve as a Function of Share Quantities



Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \ln \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

➤ Say, you buy $x \in \mathbb{R}^n$ amount

➤ You pay $V(q + x) - V(q)$; Your expected return is $\sum_{j \in [n]} \lambda_j \cdot x_j$

➤ Expected utility is

$$U(x) = \sum_{j \in [n]} \lambda_j \cdot x_j - V(q + x) + V(q)$$

➤ Which x maximizes your utility?

$$0 = \frac{\partial U(x)}{\partial x_i} = \lambda_i - \frac{\partial V(q+x)}{\partial x_i}$$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

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➤ Which x maximizes your utility?

$$0 = \frac{\partial U(x)}{\partial x_i} = \lambda_i - \frac{\partial V(q+x)}{\partial x_i} = \lambda_i - \frac{e^{(q_i+x_i)/b}}{\sum_{j \in [n]} e^{(q_j+x_j)/b}}$$

This is the market price of contract i after your purchase

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \ln \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

➤ Why non-negative?

- Buy 0 amount leads to 0, so optimal amount is at least as good

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \ln \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

- This is the expected utility you believe, but **may be incorrect since your λ may be inaccurate!**
- So, buy only when your prediction is really more accurate than the current market prediction
 - **Achieves market efficiency:** price = current best market prediction

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \ln \sum_{j \in [n]} e^{q_j/b}$

Q2: If market ends up with $q = (q_1, \dots, q_n)$ shares for each contract, how much money did the MM collect?

➤ Answer: $V(q) - V(0) = V(q) - b \ln n$

➤ But after event outcome is realized, MM need to pay based on contracts – what is the worst-case loss of MM?

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \ln \sum_{j \in [n]} e^{q_j/b}$

Fact. After event outcome realizes and MM pays the contract, worst case MM loses is $b \ln n$ (i.e., bounded).

Proof

➤ Only one event will be realized, say it is event e_i

➤ MM utility is $V(q) - b \ln n - q_i$

$$\geq b \ln e^{q_i/b} - b \ln n - q_i$$
$$\geq q_i - b \ln n - q_i$$
$$\geq -b \ln n$$

“=” can be achieved by letting $q_i \rightarrow \infty$ (i.e., people all think e_i will occur)

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Has been implemented by several prediction markets
 - E.g., InklingMarkets, Washington Stock Exchange, BizPredict, Net Exchange, and (reportedly) at YooNew.

SELECTED PREDICTION
Barack Obama

CURRENT PRICE
\$57.02

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

Do you think:

- Chances are higher than 57.02% this will occur
- Chances are lower than 57.02% this will occur

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

If you think the current odds of 57% are:

| <input type="radio"/> Way too low... | <input type="radio"/> Low... | <input type="radio"/> Just below... | <input type="radio"/> Advanced... |
|--------------------------------------|--------------------------------|-------------------------------------|-----------------------------------|
| Buy 50 shares | Buy 20 shares | Buy 5 shares | Buy <input type="text"/> shares |
| your cost \$2,971.95 | your cost \$1,159.83 | your cost \$286.30 | your cost ... |
| estimated new price \$61.84 | estimated new price \$58.97 | estimated new price \$57.51 | estimated new price ... |

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates (EWU)

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}$, $\forall i \in [n]$



Action 1, $w_t(1)$



Action 2, $w_t(2)$

...



Action n , $w_t(n)$

$$\begin{aligned}w_{t+1}(i) &= w_t(i) \cdot e^{-\epsilon c_t(i)} \\&= [w_{t-1}(i) \cdot e^{-\epsilon c_{t-1}(i)}] \cdot e^{-\epsilon c_t(i)} \\&= \dots = e^{-\epsilon C_t(i)} \text{ where } C_t(i) = \sum_{\tau \leq t} c_\tau(i)\end{aligned}$$

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}$, $\forall i \in [n]$
- At round $t + 1$, select action i with probability

$$\frac{w_t(i)}{W_t} = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

where $C_t = \sum_{\tau \leq t} c_\tau$ is the accumulated cost vector

This looks very much like the price function in LMSR (q is the accumulated sales quantity)

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

EWU vs LMSR

➤ Exponential Weight Update

- n actions
- Maintain weight $w_t(i)$
- Total cost $C_T(i) = \sum_{t \leq T} c_t(i)$
- Select i with prob

$$p_i = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

- Weights reflect how good an action is
- Care about worst case regret

$$C_T(\text{Alg}) - \min_i C_T(i)$$

➤ LMSR

- n contracts (i.e., outcomes)
- Maintain prices $p(i)$
- Total shares sold $q(i)$
- Price of contract i

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

- Prices reflect how probable is an event
- Care about worst case MM loss
(\$ received) $- \min_i q(i)$

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market markers and no-regret learning algorithms (see [[Chen and Vaughan 2010](#)])
- Markets can potentially be a very effective forecasting tool
 - Big project: “replication market” for DARPA SCORE program



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Systematizing Confidence in Open Research and Evidence (SCORE)

[Dr. Adam Russell](#)

The Department of Defense (DoD) often leverages social and behavioral science (SBS) research to design plans, guide investments, assess outcomes, and build models of human social systems and behaviors as they relate to national security challenges in the human domain. However, a number of recent empirical studies and meta-analyses have revealed that many SBS results vary dramatically in terms of their ability to be independently reproduced or replicated, which could have real-world implications for DoD's plans, decisions, and models. To help address this situation, DARPA's Systematizing Confidence in Open Research and Evidence (SCORE) program aims to develop and deploy automated tools to assign "confidence scores" to different SBS research results and claims. Confidence scores are quantitative measures that should enable a DoD consumer of SBS research to understand the degree to which a particular claim or result is likely to be reproducible or replicable. These tools will assign explainable confidence scores with a reliability that is equal to, or better than, the best current human expert methods. If successful, SCORE will enable DoD personnel to quickly calibrate the level of confidence they should have in the reproducibility and replicability of a given SBS result or claim, and thereby

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market makers and no-regret learning algorithms (see [[Chen and Vaughan 2010](#)])
- Markets can potentially be a very effective forecasting tool
 - Big project: “replication market” for DARPA SCORE program
- Mechanism design for prediction tasks
 - ML is not the only way of making predictions
 - But markets and ML may augment each other...

Thank You

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