# Announcements

- Please work on your course project
- HW3 will be posted around middle Nov

# CMSC 35401: The Interplay of Learning and Game Theory (Autumn 2022)

# Bayesian Persuasion

Instructor: Haifeng Xu

- Prediction markets and peer prediction study how to elicit information from others

- This lecture: when you have information, how to exploit it?
  - Related to manipulate features to game a learning algorithm (later lectures)

# Outline

- Introduction and Bayesian Persuasion

- Algorithms for Bayesian Persuasion

- Persuading Multiple Receivers

# Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions

You only pay the second highest bid!

# Two Primary Ways to Influence Agents’ Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons

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- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Bonus depends on performance, and is up to $1M!

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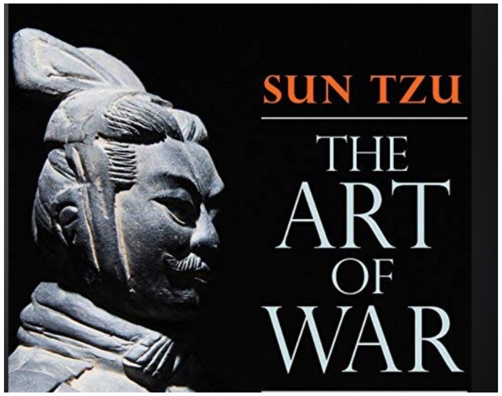
- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

} Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles

All warfare is based on deception. Hence, when we are able to attack, we must seem unable; when using our forces, we must appear inactive…

-- Sun Tzu, *The Art of War*



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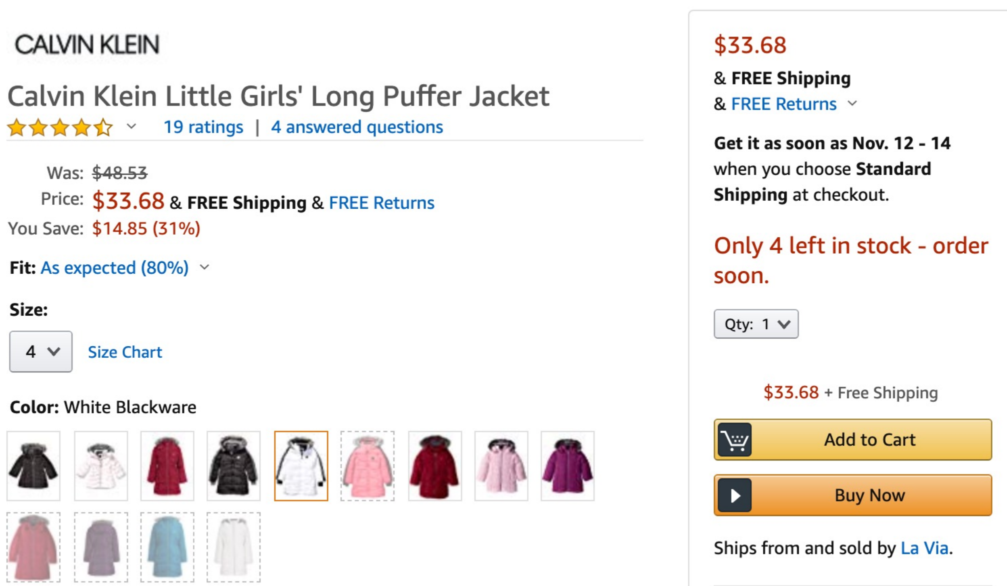
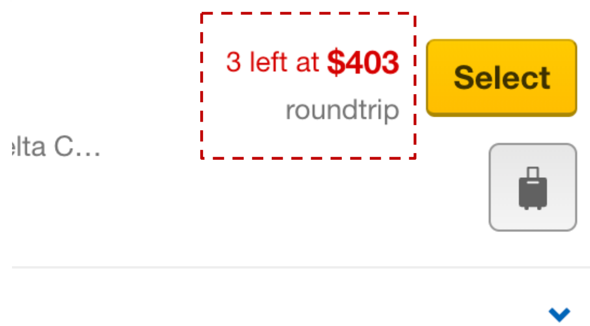
Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles
  - Strategic information disclosure

Strategic inventory information disclosure



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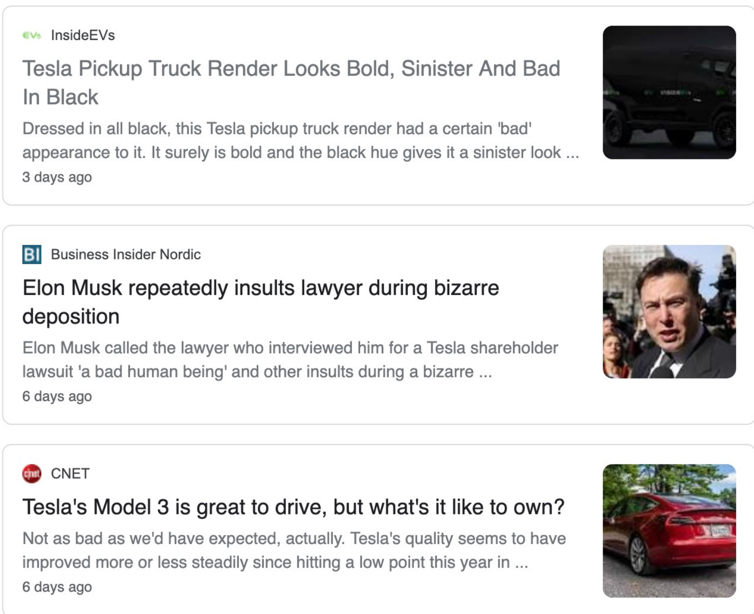
Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles
  - Strategic information disclosure
  - News articles, advertising, tweets, etc.

# Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles
  - Strategic information disclosure
  - News articles, advertising, tweets …
  - In fact, most information you see is there with a purpose

Persuasion (information design)

A whole course from Booth on this topic

Persuasion is the act of exploiting an informational advantage in order to influence the decisions of others

- Intrinsic in human activities: advertising, negotiation, politics, security, marketing, financial regulation,…
- A large body of research

**One Quarter of GDP Is Persuasion**

*By* DONALD MCCLOSKEY AND ARJO KLAMER*

— The American Economic Review Vol. 85, No. 2, 1995.

# Example: Recommendation Letters


- Advisor vs. recruiter
- 1/3 of the advisor’s students are excellent; 2/3 are average
- A fresh graduate is randomly drawn from this population
- Recruiter
  - Utility $1 + \epsilon$ for hiring an excellent student; $-1$ for an average student
  - Utility $0$ for not hiring
  - A-priori, only knows the advisor’s student population

$$(1+\epsilon)\times 1/3 - 1\times 2/3 \qquad < \qquad 0$$

*hiring* *Not hiring*

# Example: Recommendation Letters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  - Utility $0$ for not hiring
  - A-priori, only knows the advisor's student population
- Advisor
  - Utility 1 if the student is hired, 0 otherwise
  - Knows whether the student is excellent or not

# Example: Recommendation Letters


What is the advisor's optimal "recommendation strategy"?

- Attempt 1: always say "excellent" (equivalently, no information)
  - Recruiter ignores the recommendation
  - Advisor expected utility 0

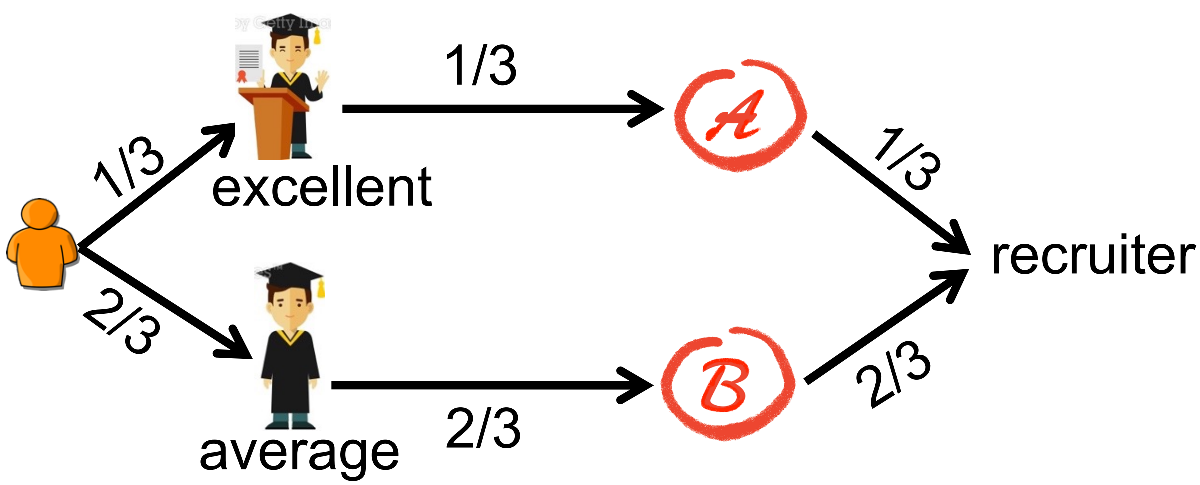
**Remark**

Assume advisor "commits" to some policy, and recruiter is fully aware this policy and will best respond

# Example: Recommendation Letters


What is the advisor’s optimal “recommendation strategy”?

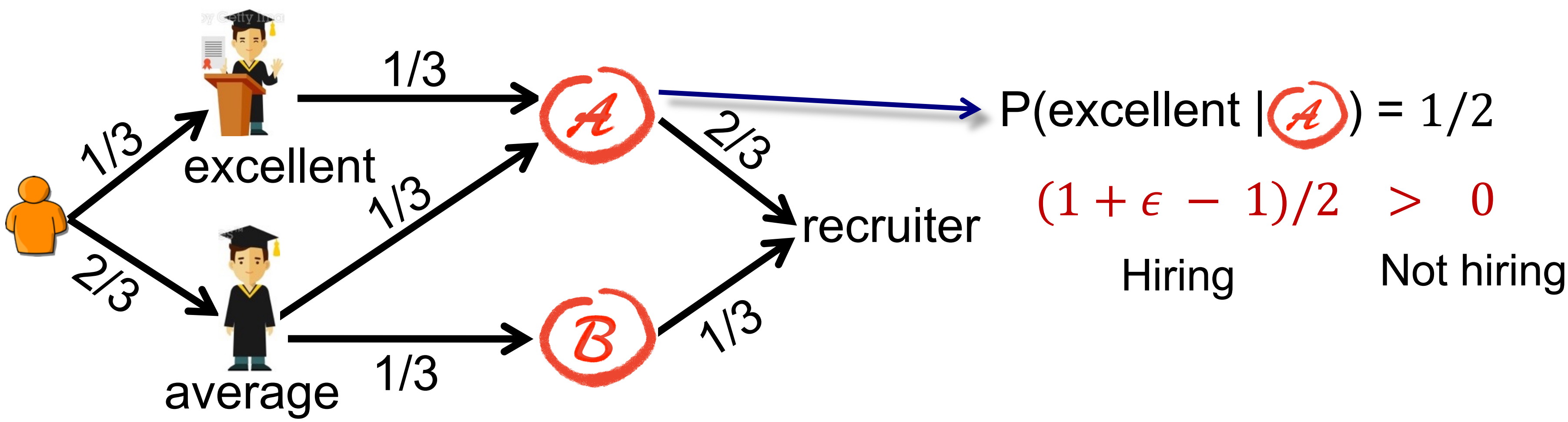
- Attempt 2: honest recommendation (i.e., full information)
  - Advisor expected utility 1/3



# Example: Recommendation Letters


What is the advisor's optimal "recommendation strategy"?

- Attempt 3: noisy information → advisor expected utility 2/3



# Model of Bayesian Persuasion

- Two players: persuader (Sender, she), decision maker (Receiver he)
  - Previous example: advisor = sender, recruiter = receiver
- Receiver looks to take an action $i \in [n] = \{1, 2, \ldots, n\}$
  - Receiver utility $r(i, \theta)$
  - Sender utility $s(i, \theta)$

  $\theta \in \Theta$ is a random state of nature
- Both players know $\theta \sim prior\ dist.\ \mu$, but Sender has an informational advantage – she can observe realization of $\theta$
- Sender wants to strategically reveal info about $\theta$ to "persuade" Receiver to take an action she likes
  - Concealing or revealing all info is not necessarily the best

Well…how to reveal partial information?

# Revealing Information via Signaling

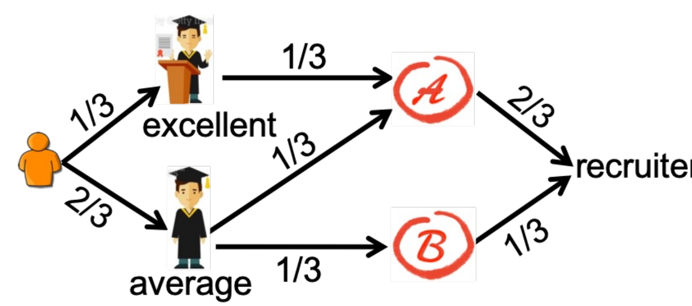
**Definition**: A signaling scheme is a mapping $\pi: \Theta \rightarrow \Delta_{\Sigma}$ where $\Sigma$ is the set of all possible signals.

$\pi$ is fully described by $\{\pi(\sigma, \theta)\}_{\theta \in \Theta, \sigma \in \Sigma}$ where $\pi(\sigma, \theta) =$ prob. of sending $\sigma$ when observing $\theta$ (so $\sum_{\sigma \in \Sigma} \pi(\sigma, \theta) = 1$ for any $\theta$)

Note: scheme $\pi$ is always assumed public knowledge, thus known by Receiver

Example

- $\Theta = \{Excellent, Average\}$, $\Sigma = \{A, B\}$
- $\pi(A, Average) = 1/2$



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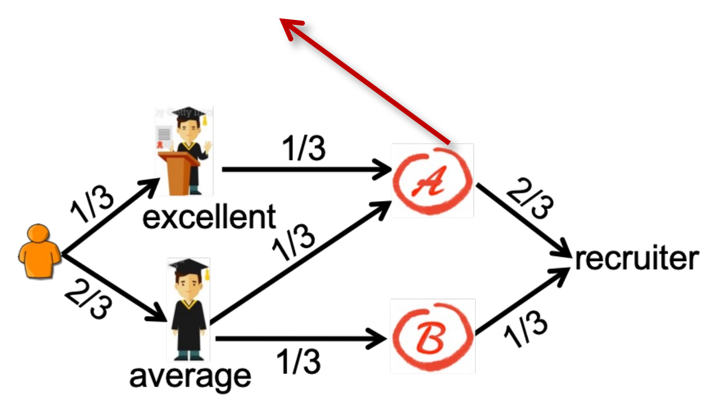
What can Receiver infer about $\theta$ after receiving σ?

Bayes updating:

$$\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'}\pi(\sigma,\theta')\cdot\mu(\theta')}$$

$\Pr(excellent|A) = 1/2$



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Would such noisy information benefit Receiver?

- Expected Receiver utility conditioned on $\sigma$:

$$R(\sigma) = \max_{i \in [n]} \left[ \sum_{\theta \in \Theta} r(i, \theta) \cdot \frac{\pi(\sigma,\theta) \cdot \mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta') \cdot \mu(\theta')} \right]$$

- $\Pr(\sigma) = \sum_{\theta'} \pi(\sigma, \theta') \cdot \mu(\theta')$

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- $\Pr(\sigma) = \sum_{\theta'} \pi(\sigma, \theta') \cdot \mu(\theta')$
  - $\Pr(\sigma) \cdot R(\sigma) = \max_i \sum_{\theta\in\Theta} r(i, \theta) \cdot \pi(\sigma, \theta) \cdot \mu(\theta)$ (a linear function of $\pi$)
- Expected Receiver utility under $\pi$: $\sum_\sigma \Pr(\sigma) \cdot R(\sigma)$

# Revealing Information via Signaling

**Fact**. Receiver's expected utility (weakly) increases under any signaling scheme $\pi$.

Proof:

- Expected Receiver utility without information: $\max_i \sum_{\theta \in \Theta} r(i,\theta) \cdot \mu(\theta)$

- Expected Receiver utility under $\pi$: $\sum_{\sigma} \Pr(\sigma) \cdot R(\sigma)$

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$$\sum_\sigma \Pr(\sigma) \cdot R(\sigma) = \sum_\sigma \Pr(\sigma) \cdot \max_{i \in [n]} \left[\sum_{\theta \in \Theta} r(i, \theta) \cdot \frac{\pi(\sigma, \theta) \cdot \mu(\theta)}{\Pr(\sigma)}\right]$$

$$\geq \max_{i \in [n]} \sum_\sigma \Pr(\sigma) \cdot \left[\sum_{\theta \in \Theta} r(i, \theta) \cdot \frac{\pi(\sigma, \theta) \cdot \mu(\theta)}{\Pr(\sigma)}\right]$$

By HW2 problem 1

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$$= \max_{i\in[n]} \sum_{\theta\in\Theta} r(i,\theta) \cdot \left(\sum_\sigma \pi(\sigma,\theta)\right) \cdot \mu(\theta)$$

$$= \max_{i\in[n]} \sum_{\theta\in\Theta} r(i,\theta) \cdot \mu(\theta)$$

# Revealing Information via Signaling

**Fact**. Receiver's expected utility (weakly) increases under any signaling scheme $\pi$.

Remarks:

- Signaling scheme does increase Receiver's utility
- More (even noisy) information always helps a decision maker (DM)
  - Not true if multiple DMs (will see examples later)

**Corollary**. Receiver's expected utility is maximized when Sender reveals full info, i.e., directly revealing the realized $\theta$.

Because any other noisy scheme $\pi$ can be improved by further revealing $\theta$ itself

# Revealing Information via Signaling

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**Corollary**. Receiver's expected utility is maximized when Sender reveals full info, i.e., directly revealing the realized $\theta$.

But this is not Sender's goal…

**Sender Objective**: maximize her own expected utility by picking $\pi$

# Outline

- Introduction and Bayesian Persuasion
- Algorithms for Bayesian Persuasion
- Persuading Multiple Receivers

**Q**: What are obstacles when designing $\pi = \{\pi(\theta, \sigma)\}_{\theta \in \Theta, \sigma \in \Sigma}$?

- Don't know what is the set of all possible signals $\Sigma$...
  - Too many signals in this world to choose from (think about how many ways Amazon can reveal information to you)
- Key observation: a signal is mathematically nothing but a posterior distribution over $\Theta$
  - Recall the Bayes updates: $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta) \cdot \mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta') \cdot \mu(\theta')}$
- It turns out that $n$ signals suffice

# Revelation Principle

**Fact**. There always exists an optimal signaling scheme that uses at most $n$(= # receiver actions) signals, where signal $\sigma_i$ induce optimal Receiver action $i$

- Conditioned on any signal $\sigma$
  - Receiver infers $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'}\pi(\sigma,\theta')\cdot\mu(\theta')}$
  - Receiver takes optimal action $i^* = \arg\max_{i\in[n]} \sum_\theta \Pr(\theta|\sigma)\ r(i,\theta)$
- If two signal $\sigma$ and $\sigma'$ result in the same best action $i^*$, Sender can combine them as a single signal $\sigma_{i^*} = (\sigma,\sigma')$
  - Claim: $i^*$ is still the optimal action conditioned on $\sigma_{i^*}$

$$\sum_\theta \Pr(\theta|\sigma)\ r(i^*,\theta) \geq \sum_\theta \Pr(\theta|\sigma)\ r(i,\theta),\ \ \forall i \qquad \times p = \Pr(\sigma)$$

$$\sum_\theta \Pr(\theta|\sigma')\ r(i^*,\theta) \geq \sum_\theta \Pr(\theta|\sigma')\ r(i,\theta),\ \ \forall i \qquad \times q = \Pr(\sigma')$$

$$\Rightarrow \quad \sum_\theta[\Pr(\theta|\sigma)p + \Pr(\theta|\sigma')q]r(i^*,\theta)$$
$$\geq \sum_\theta[\Pr(\theta|\sigma)p + \Pr(\theta|\sigma')q]r(i,\theta),\quad \forall i$$

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$$\sum_\theta \Pr(\theta|\sigma)\, r(i^*,\theta) \geq \sum_\theta \Pr(\theta|\sigma)\, r(i,\theta), \quad \forall i$$

$$\sum_\theta \Pr(\theta|\sigma')\, r(i^*,\theta) \geq \sum_\theta \Pr(\theta|\sigma')\, r(i,\theta), \quad \forall i$$

$$\Rightarrow \quad \sum_\theta [\Pr(\theta|\sigma)p + \Pr(\theta|\sigma')q]\, r(i^*,\theta) \geq \sum_\theta [\Pr(\theta|\sigma)p + \Pr(\theta|\sigma')q]\, r(i,\theta), \quad \forall i$$

$\Pr(\theta|\sigma_{i^*})$ is a linear combination of $\Pr(\theta|\sigma)$ and $\Pr(\theta|\sigma')$

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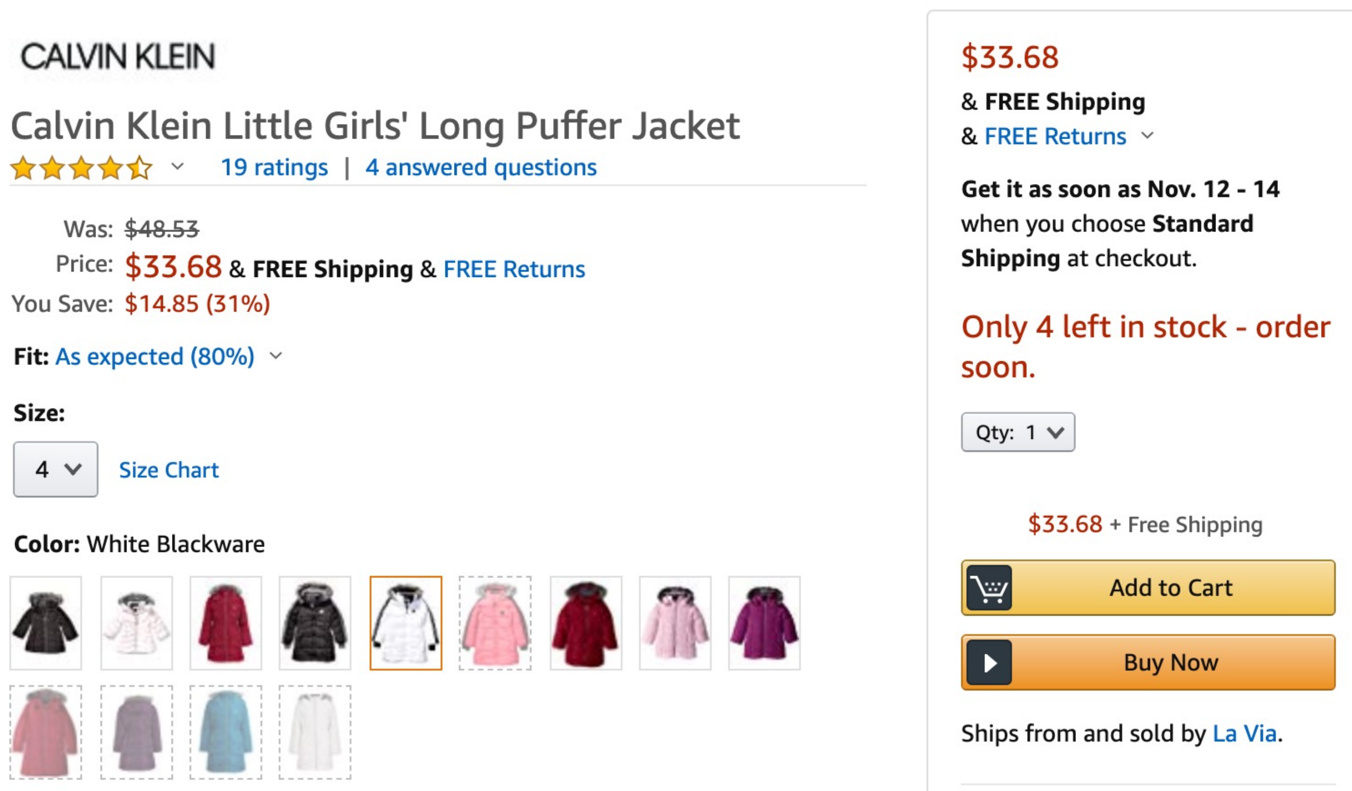
- Conditioned on any signal $\sigma$
  - Receiver infers $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta')\cdot\mu(\theta')}$
  - Receiver takes optimal action $i^* = \arg\max_{i\in[n]} \sum_\theta \Pr(\theta|\sigma)\, r(i,\theta)$
- If two signal $\sigma$ and $\sigma'$ result in the same best action $i^*$, Sender can combine them as a single signal $\sigma_{i^*} = (\sigma, \sigma')$
  - Claim: $i^*$ is still the optimal action conditioned on $\sigma_{i^*}$
  - Both players' utilities did not change as receiver still takes $i^*$ as Sender wanted
- Can merge all signals with optimal receiver action $i^*$ as a single signal $\sigma_{i^*}$

# Revelation Principle

**Fact**. There always exists an optimal signaling scheme that uses at most $n$(= # receiver actions) signals, where signal $\sigma_i$ induce optimal Receiver action $i$

- Each $\sigma_i$ can be viewed as an action recommendation of $i$



# Optimal Persuasion via Linear Program

- Input: prior $\mu$, sender payoff $s(i,\theta)$, receiver payoff $r(i,\theta)$
- Variables: $\pi(\sigma_i,\theta)$

Sender expected utility
(we know Receiver will take $i$ at signal $\sigma_i$)

$$
\begin{aligned}
\max \quad & \boxed{\textstyle\sum_{\theta\in\Theta}\sum_{i=1}^{n} s(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta)} & \\
\text{s.t.} \quad & \textstyle\sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) \geq \sum_{\theta\in\Theta} r(j,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta), & \text{for } i,j\in[n]. \\
& \textstyle\sum_{i=1}^{n}\pi(\sigma_i,\theta) = 1, & \text{for } \theta\in\Theta. \\
& \pi(\sigma_i,\theta)\geq 0, & \text{for } \theta\in\Theta, i\in[n].
\end{aligned}
$$

# Optimal Persuasion via Linear Program

- Input: prior $\mu$, sender payoff $s(i,\theta)$, receiver payoff $r(i,\theta)$
- Variables: $\pi(\sigma_i,\theta)$

$\sigma_i$ indeed incentivizes Receiver best action $i$

$$
\begin{array}{lll}
\max & \sum_{\theta\in\Theta}\sum_{i=1}^{n} s(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) & \\
\text{s.t.} & \sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) \geq \sum_{\theta\in\Theta} r(j,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta), & \text{for } i,j\in[n]. \\
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$$

$\pi$ is a valid signaling scheme

# Optimal Persuasion via Linear Program

- Input: prior $\mu$, sender payoff $s(i,\theta)$, receiver payoff $r(i,\theta)$
- Variables: $\pi(\sigma_i,\theta)$

$$
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\end{aligned}
$$

This should remind you the LP for correlated equilibria

# Outline

- Introduction and Bayesian Persuasion
- Algorithms for Bayesian Persuasion
- Persuading Multiple Receivers

# Recommendation Letter Example cont'd

- Advisor vs. two fellowship programs
- 1/3 of the advisor's students are excellent; 2/3 are average
- A fresh graduate is randomly drawn from this population
- Each fellowship:
  - Utility $1 + \epsilon$ for awarding excellent student; $-1$ for average student
  - Utility $0$ for no award
  - A-priori, only knows the advisor's student population
  - Student can accept both fellowships
- Advisor
  - Utility 1 if student gets at least one fellowship, 0 otherwise
  - Knows whether the student is excellent or not

# Recommendation Letter Example cont'd

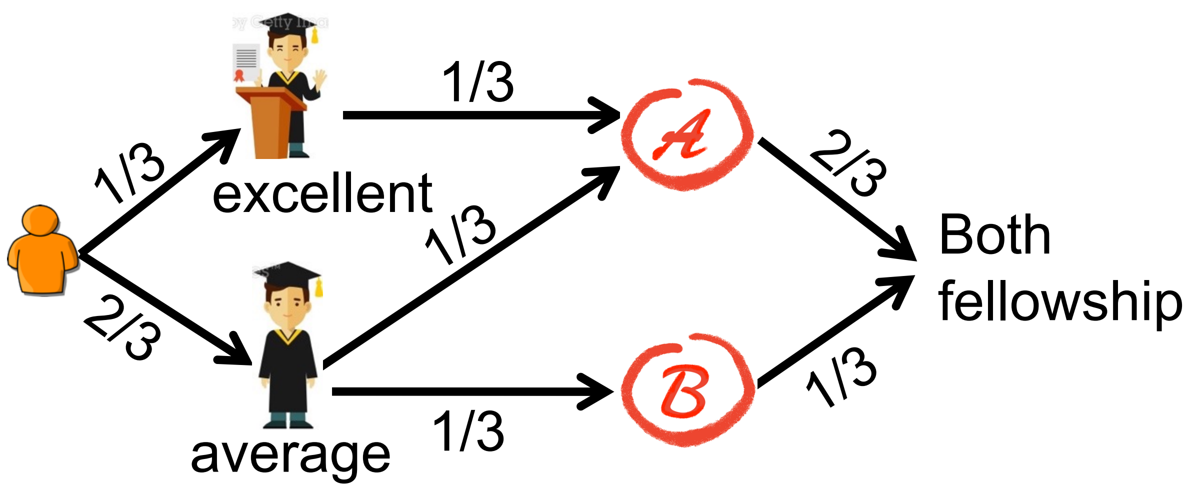
What is the advisor's optimal "recommendation strategy"?

Well, we learned the lesson — noisy info!

# Recommendation Letter Example cont'd

What is the advisor's optimal "recommendation strategy"?

➢ Optimal public scheme → advisor expected utility 2/3



# Recommendation Letter Example cont'd

What is the advisor's optimal "recommendation strategy"?

- Optimal private scheme → advisor expected utility 1

excellent

# Recommendation Letter Example cont'd

What is the advisor's optimal "recommendation strategy"?

- Optimal private scheme → advisor expected utility 1

- Conditioned on "strong", excellent with prob ½
- Always at least one fellowship recommended "strong"


average

# Recommendation Letter Example cont'd

Generalize this example to $n$ fellowships:

advisor utility of optimal private scheme

$\geq \frac{n+1}{2}$ advisor utility of optimal pubic scheme

**Conceptual Message**

Being able to persuade privately may have a huge advantage

Remark: fellowship programs' utilities did not decrease

# Thank You

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