CMSC 35401: The Interplay of Economics and ML (Winter 2024)

Introduction

Instructor: Haifeng Xu

Outline

- Course Overview
- Administrivia
- An Example

Single-Agent Decision Making

- A decision maker picks an action $x \in X$, resulting in utility $f(x)$
- Typically an optimization problem:

$$\begin{array}{ll} \text{minimize (or maximize)} & f(x) \\ \text{subject to} & x \in X \end{array}$$

  - x: decision variable
  - $f(x)$: objective function
  - X: feasible set/region
  - Optimal solution, optimal value

- Example 1: minimize x^2, s.t. $x \in [-1,1]$

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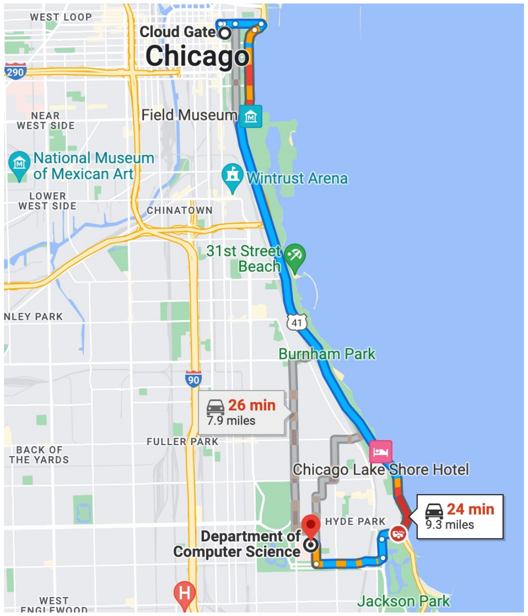

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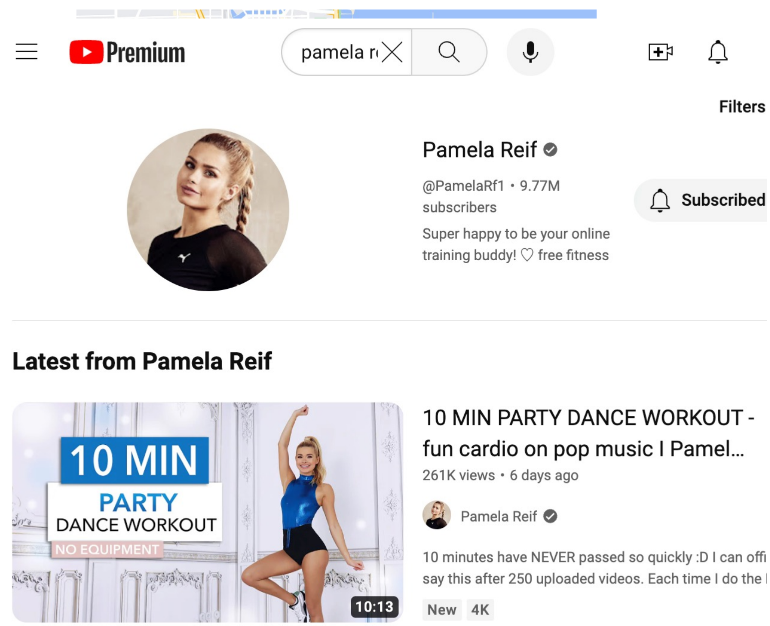
- Example 2: pick a road to school
- Example 3: build a Youtube channel



Multi-Agent Decision Making

- Usually, your payoffs affected not only by your actions, but also others'
- Agent i's utility $f_i(x_i, x_{-i})$ depends on his own action x_i, as well as other agents' actions x_{-i}
- Is this still an optimization problem? Should each agent i just pick $x_i \in X_i$ to minimize $f_i(x_i, x_{-i})$?
  - x_{-i} is not under i's control
  - Think of rock-paper-scissor game
- Examples: build a Youtube channel, routing, sales, even taking courses…

Example I: Prisoner's Dilemma

- Two members A,B of a criminal gang are arrested
- They are questioned in two separate rooms
  - No communications between them

B / A	B stays silent	B betrays
A stays silent	-1 $\boxed{-1}$	0 -3
A betrays	-3 $\boxed{0}$	-2 -2

Q: How should each prisoner act?

- Betray is always the best action

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  - ❖ No communications between them

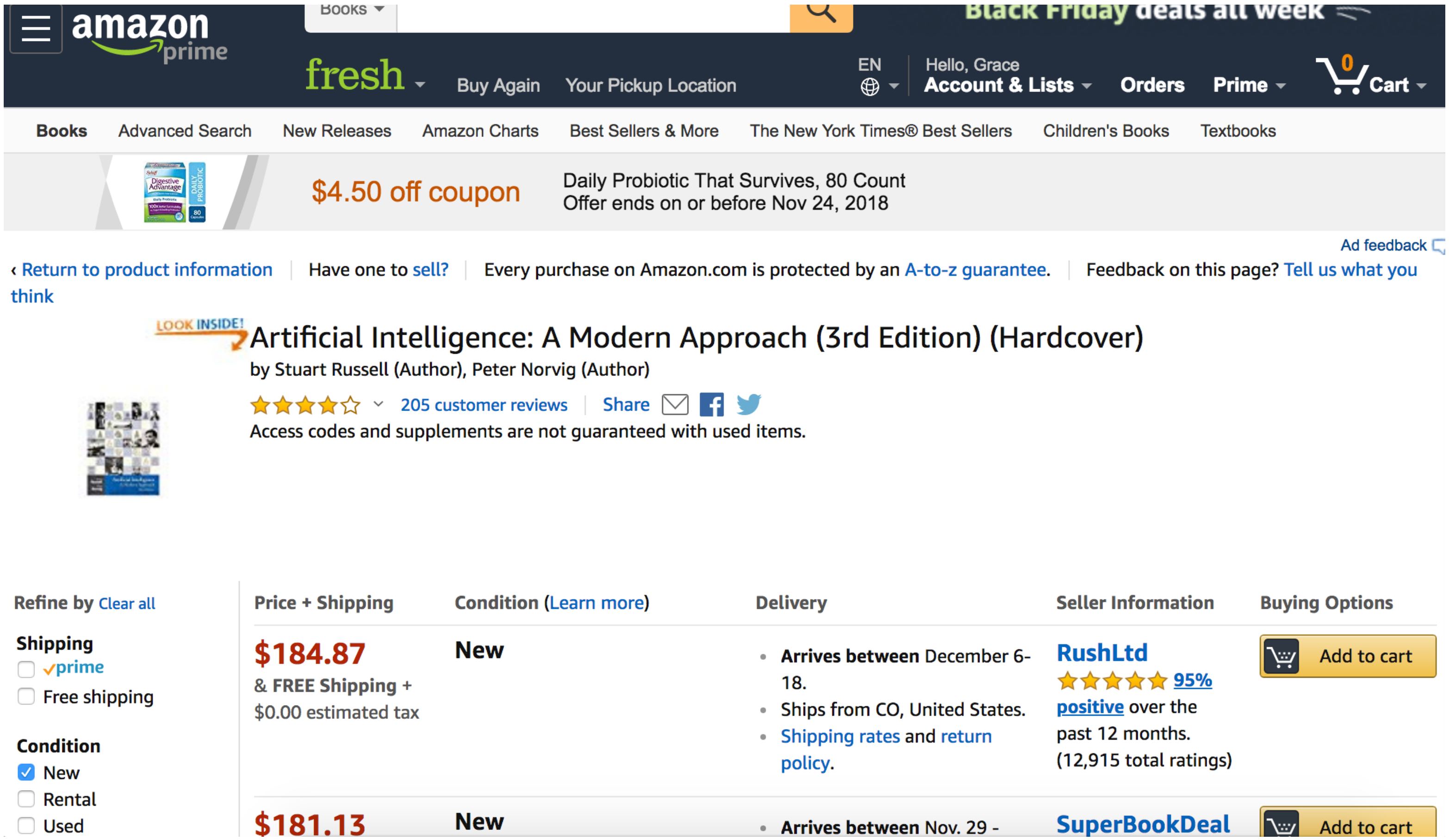
B A	B stays silent	B betrays
A stays silent	-1 -1	0 -3
A betrays	-3 0	$\boxed{-2 \quad -2}$

equilibrium

Q: How should each prisoner act?

- ➢ Betray is always the best action
- ➢ But, (-1,-1) is a better outcome for both
- ➢ Why? What goes wrong?
  - • Selfish behaviors lead to inefficient outcome

Example II: Markets on Amazon

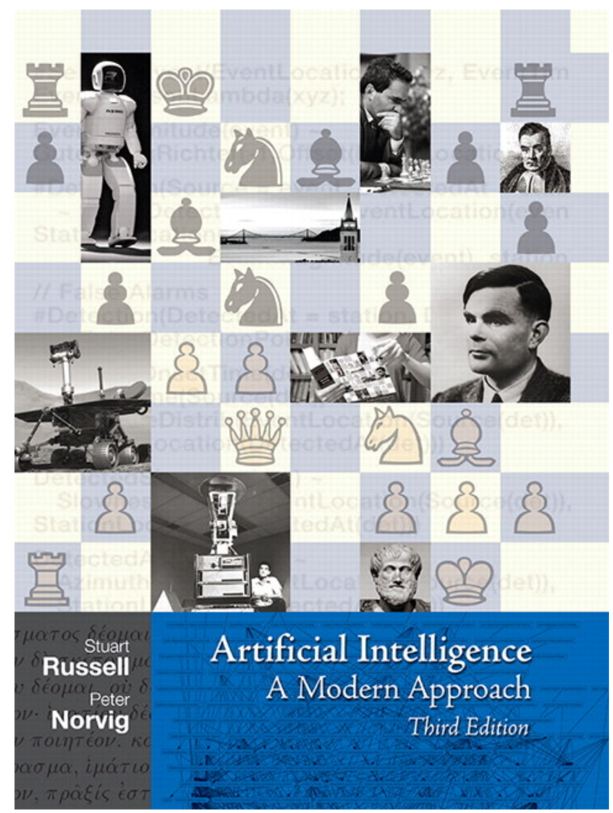


Example II: Markets on Amazon

- Assume people will buy if the book price ≤ $200
- Product cost = $20

If the market has only one book seller…

Q: What price should this monopoly set?



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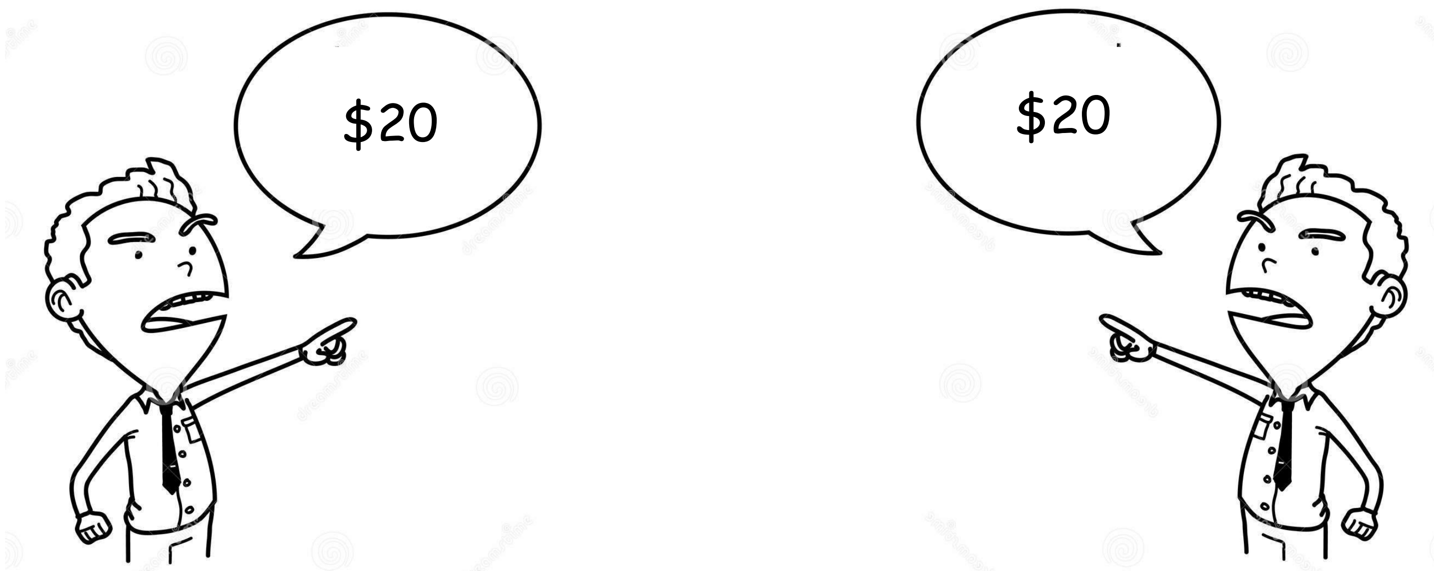

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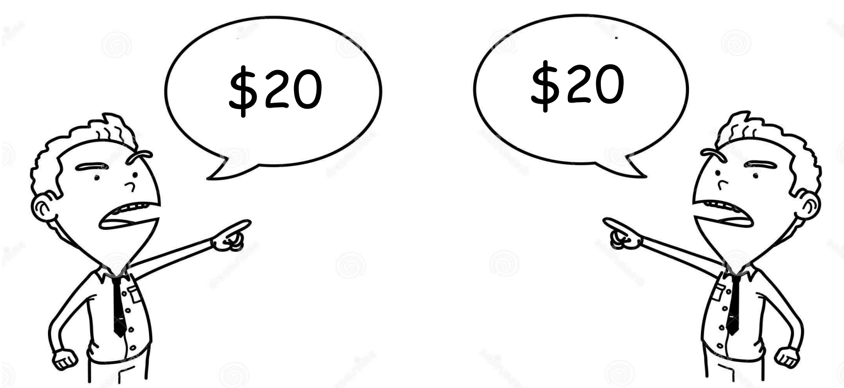
What if the market has two book sellers…

Q: What price should each seller set?

- The market reaches a "stable status" (a.k.a., equilibrium)
- Nobody can benefit via *unilateral deviation*



- Bertrand competition
- Seller's revenue-maximizing behaviors lead to low revenue

Economic Analysis and Game Theory

Game Theory studies economic/multiple-agent decision making in scenarios where an agent's payoff depends on other agents' actions.

- Fundamental concept --- Equilibrium
  - A "stable status" at which any agent cannot improve his payoff through unilateral deviation
  - A solution concept (i.e., outcome) used to describe the system
  - Resembles "optimal decision" in single-agent case
- A central theme in game theory is to study the equilibrium
  - Different "types" of equilibria
  - May not exist; even exist, not necessarily unique
  - Understand properties of equilibrium, compute equilibria, how to improve inefficiency of equilibrium . . .

Machine Learning

- Difficult to give a universal definition
- At a high level, the task is to learn a function $f: X \rightarrow Y$, where $(\mathrm{x}, \mathrm{y}) \in X \times Y$ is drawn from some distribution D
  - Input: a set of samples $\{(x_i, y_i)\}_{i=1,2,\cdots,n}$ drawn from D
  - Output: an algorithm $A: X \rightarrow Y$ such that $A(x) \approx f(x)$ (usually measured by some loss function)
- Examples
  - Classification: $X =$ feature vectors; $Y = \{0,1\}$
  - Regression: $X =$ feature vectors; $Y = \mathbb{R}$
  - Reinforcement learning has a slightly different setup, but can be thought as $X =$ state space, Y = action space

Problems at Interface of Learning and Game Theory

- If a game is unknown or too complex, can players learn to play the game optimally?
  - Yes, sometimes – no regret learning and convergence to equilibrium
- Can game-theoretic models inspire machine learning models?
  - Yes, GANs which are zero-sum games
- Data is the fuel for ML – can we quantify economic value of data?
  - Yes, using ideas from coalitional game theory
- We know how to learn to recognize faces or languages, but can we also learn the design of games to achieve some goal?
  - Yes, learning optimal auctions, product pricing schemes, etc
- Gaming behaviors in ML? How to handle them? Societal impact?
  - Yes, e.g, learn whether to give loans to someone or whether to admit a student to Uchicago based on their features
- . ..

Goodhart's Law

When a measure becomes a target, it ceases to be a good measure

Main Topics of This Course

First Half: Machine learning for economic problems

- Basics of linear programming and game theory
- Online learning and its convergence to equilibrium

Second Half: Economic aspects of machine learning

- Economic principles for the valuation and pricing of data
- Handle gaming behaviors in machine learning
  - Particularly, algorithms, fairness, societal impacts
- The economy of online content creation and new challenges under generative AI

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Only cover fundamentals of each direction

Main Topics of This Course

	PART I:	**Learning for Economic Problems**
1 (Jan 4: I)	Introduction [slides]	Kleinberg/Leighton paper
2 (Jan 4: II)	Basics of LPs [slides]	Chapter 2.1, 2.2, 4.3 of Convex Optimization by Boyd and Vandenberghe
3 (Jan 11: I)	LP duality [slides]	Lecture notes 5 and 6 of an optimization course by Trevisan
4 (Jan 11: II)	Intro to Game Theory (I) [slides]	Section 3.1, 3.2, 3.3 of an game theory book by Shoham and Leyton-Brown
5 (Jan 18: I)	Intro to Game Theory (II) [slides]	Equilibrium analysis of GANs by Arora et al.
6 (Jan 18: II)	Intro to Online Learning [slides]	
7 (Jan 25: I)	Multiplicative Weight [slides]	A survey paper on MWU and its applications by Arora et al.
8 (Jan 25: II)	Swap Regret [slides]	A note by Balcan on converting regret to swap regret
9 (Feb 1: I)	Multi-Armed Bandits [slides]	Section 2, 3 of the Book by Bubeck and Cesa-Bianch on Bandits
	PART II:	**Economic Aspects of Machine Learning**
10 (Feb 1: II)	Information Design [slides]	Bayesian Persuasion and Information Design paper
11 (Feb 8: I)	Valuation and Pricing of Information	Quantifying information and Optimal Pricing of Information
12 (Feb 8: II)	Shapley Value, Data Valuation	Shapley's original paper and its applications to valuating data
13 (Feb 15: I)	Strategic Learning I	PAC-learning for Strategic Classification paper
14 (Feb 15: II)	Strategic Learning II	How Can ML Induce Right Efforts paper
15 (Feb 22: I)	Tradeoffs of Fairness	Inherent Trade-Offs in the Fair Determination
16 (Feb 22: II)	Performative Prediction	Performative Prediction: Past and Future
17 (Feb 29: I)	Economics of Generative AI	Mechanism Design for Large Language Models
18 (Feb 29: II)	Project presentations	

Course Goal

- Get familiar with basics of economic principles and learning
- Understand machine learning questions in economic settings, and how to deal with some of them
- Understand the value of data, online contents, recommendation
- Aware of gaming behaviors in machine learning applications, and how to deal with some of them
- Can understand cutting-edge research papers in relevant areas

Targeted Audience of This Course

- Anyone planning to do research at the interface of economics (or algorithm design) and machine learning
  - This is a new research direction with many opportunities/challenges
  - Recent breakthrough in no-limit poker is an example

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- Anyone planning to do research at the interface of economics (or algorithm design) and machine learning
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  - Recent breakthrough in no-limit poker is an example
- Anyone interested in theoretical ML, strategic reasoning, human factors in learning, AI
  - As more and more ML systems interact with human beings, such strategic reasoning becomes increasingly important
  - With more techniques developed for ML, they also broadened our toolkits for designing and solving games
- Anyone interested in understanding basics of economics and learning

Who May not Be Suitable for This Course?

- Those who do not satisfy the prerequisites "in practice"
- Those who are looking for a recipe to implement ML/DL algorithms, or want to learn how to use TensorFlow, PyTorch, etc.
  - This is primarily a theory course
  - We will mostly focus on simple/basic yet theoretically insightful problems
  - The course is proof based – we will not write code

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Basic Information

- Course time: Thursday, 2:00 pm – 4:50 pm, with 15 mins break at the middle
- Lecture: in person (unless further instruction)
- Instructor: Haifeng Xu
  - Email: haifengxu@uchicago.edu
  - Office Hour: 4:50 to 5:50 pm Thur (rightly after class)
  - Can add more office hour, depending on demand
- TAs
  - **No TA** curently
- Couse website: www.haifeng-xu.com/cmsc35401win24/index.htm
  - Easier way is to search my personal website and navigates to course
- References: linked papers/notes on website, no official textbooks
  - Slides will be posted *after* lecture

Prerequisites

- Mathematically mature: be comfortable with proofs
- Sufficient exposures to probabilities and algorithms/optimization
  - CMSC 27200/27220 and equivalent
  - We will cover basics of optimization

Requirements and Grading

- Part I: 10% participation
- Part II: research project, 45% of grade. Project instructions will be posted on website later.
  - Team up: 2 – 4 people per team
  - Raise *novel* technical questions and provide some *nontrivial* answers
  - Deliverables: a presentation + a technical report in PDF
  - Grading is based on novelty + non-triviality

Requirements and Grading

- Part III: 3~4 homework, 45% of grade.
  - Proof based
  - Discussion allowed, even encouraged, but must write up solutions independently
  - Must be written up in Latex – hand-written solutions will not be accepted
  - One late homework allowed, at most 2 days
- Taking for electives
  - Need to additionally complete bonus questions (often more challenging) in each HW
  - HW still counts for 45%
- FYI: no need to worry about your grade if you do invest time

If you have any suggestions/comments/concerns, feel free to email me.

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Learning to Sell a Product

- You are a product seller facing N unknown buyers
- These buyers all value your product at the same $v \in [0,1]$, which however is *unknown* to you
- Buyers come in sequence $1,2,\cdots,N$; For each buyer, you can choose a price p and ask him whether he is willing to buy the product
  - If $v \geq p$, she/he purchases; otherwise leaves the queue

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- Buyers come in sequence $1,2,\cdots,N$; For each buyer, you can choose a price p and ask him whether he is willing to buy the product
  - If $v \geq p$, she/he purchases; otherwise not
- How to quickly learn these buyers' value v within precision $\epsilon = \frac{1}{N}$?
  - This is a pure learning problem
  - (Well, you may directly ask a buyer's value, but guess what will happen?)
- Answer: $\log(N)$ rounds via BinarySearch

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Let us move to a natural game-theoretic setup ……

- You have an ultimate objective of maximizing your revenue, but do not really care about learning v (though you may have to)
- How much revenue can BinarySearch secure?
  - May get really unlucky in first $\log(N)$ rounds and no sale happened
  - After $\log(N)$ rounds, can set a price $p \geq \tilde{v} - 1/N$ ($\tilde{v}$ is learned value)

$$\text{Rev} = \underbrace{0}_{\text{First } \log(N) \text{ rounds}} + \underbrace{(N - \log N)(v - \frac{2}{N})}_{\text{Remaining rounds}} \approx vN - v\log N \text{ -2}$$

Regret as Performance Measure

- To measure algorithm performance, we use regret

Regret $:=$ how much less is an algorithm's utility compared to the (idealized) case where we know v.

- Had we know v, should just price the product at $p = v$, earning vN
- The regret is then

$$\text{Regret(binary search)} \approx vN - [vN - v\log N - 2] = v\log N + 2$$

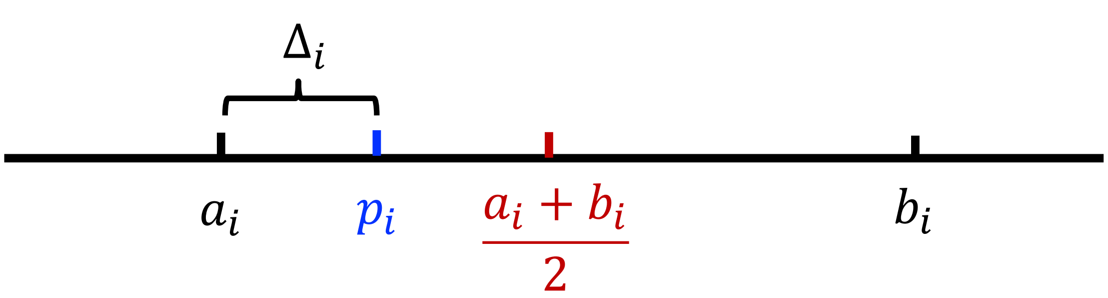
Q: Is this the best (i.e., the smallest) regret?

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2 \log \log N)$

Why BinarySearch may be bad?

- For buyer i, BinarySearch maintains an interval bound $[a_i, b_i]$ and use $p_i = (a_i + b_i)/2$ for buyer i
  - This learns v as quickly as possible
  - But maybe bad for revenue since we will get 0 revenue if $p_i > v$, and $p_i = (a_i + b_i)/2$ may be too high/aggressive



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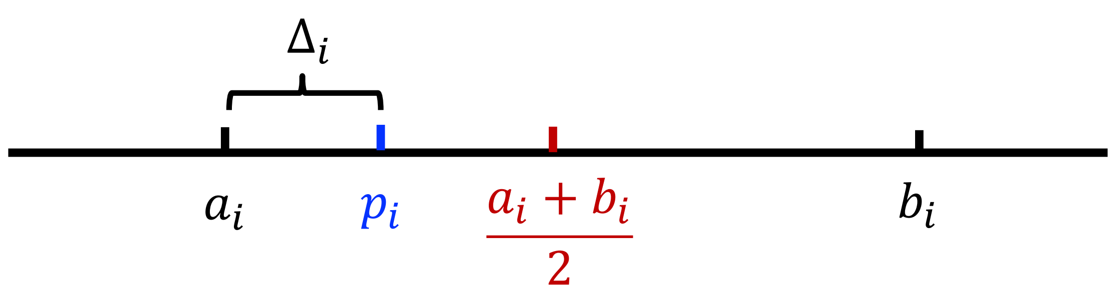
- Algorithm idea: use more conservative prices



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The Algorithm (note $v \in [0,1]$):

- Maintains an interval bound $[a_i, b_i]$ and a step size Δ_i
- Offer price $p_i = a_i + \Delta_i$ for buyer i

- If i accepts, update $a_{i+1} = p_i$, $b_{i+1} = b_i$, $\Delta_{i+1} = \Delta_i$

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- Otherwise, update $a_{i+1} = a_i$, $b_{i+1} = p_i$, $\Delta_{i+1} = (\Delta_i)^2$
- Start with $a_1 = 0$, $b_1 = 1$, $\Delta_1 = 1/2$; Once $b_i - a_i \le \frac{1}{N}$, always use $p = a_i$ afterwards

Remark: searching smaller region with smaller step size.

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2\log\log N)$

Algorithm analysis:

Claim 1: The step size Δ_i takes values 2^{-2^j} for $j = 0, 1, \cdots$. Moreover, whenever $\Delta_{i+1} = (\Delta_i)^2$ happens, $b_{i+1} - a_{i+1} = \sqrt{\Delta_{i+1}}$.

Proof

- Recall $\Delta_1 = \frac{1}{2} = 2^{-2^0}$, and step size update $\Delta_{i+1} = (\Delta_i)^2$
- If $\Delta_i = 2^{-2^j}$, then $(\Delta_i)^2 = 2^{-2^j - 2^j} = 2^{-2^{j+1}}$
- When $\Delta_{i+1} = (\Delta_i)^2$ happens, $b_{i+1} - a_{i+1} = \Delta_i = \sqrt{\Delta_{i+1}}$

An Algorithm with Smaller Regret

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Algorithm analysis:

- After $b_i - a_i \leq \frac{1}{N}$, the total regret is at most 1
  - Because (1) regret of each step is at most $\frac{1}{N}$; (2) there are at most N rounds
- Main step is to bound regret before reaching $b_i - a_i = \frac{1}{N}$

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2\log\log N)$

Algorithm analysis:

- How many step size value updates needed to reach $b_i - a_i = \frac{1}{N}$?
  - $\log\log N$: set $2^{-2^i} = \frac{1}{N}$ → $i = \log\log N$
  - The following claim then completes the proof of the theorem

Claim 2: total regret from any step size value Δ is at most 2.

- No sale happens only once for any step size → regret at most 1

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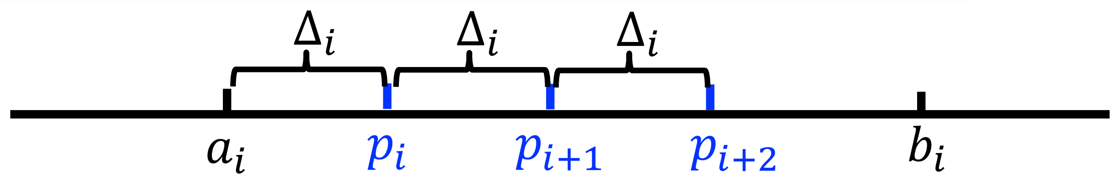

➢ How many step size value updates needed to reach $b_i - a_i = \frac{1}{N}$?

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Claim 2: total regret from any step size value Δ is at most 2.

➢ No sale happens only once for any step size → regret at most 1

➢ What about the regret when sales happen?

- Can happen at most $\sqrt{\Delta}/\Delta$ times since $b_i - a_i \leq \sqrt{\Delta}$; regret from each time is at most $b_i - a_i (\leq \sqrt{\Delta})$
- Regret from sales is at most $(\sqrt{\Delta}/\Delta) \times \sqrt{\Delta} = 1$

An Algorithm with Smaller Regret

Remarks

- $O(\log \log N)$ is also the order-wise best regret [KL, FOCS'13]
- This is an example of exploration vs exploitation
  - Exploration: want to learn v
  - Exploitation: but ultimate goal is to utilize learned v to maximize revenue
  - More in later lectures…
- $\mathsf{BinarySearch}$ is best for exploration, but did not balance the two
- The "optimal" algorithm uses less step value updates, but more interval updates
  - Less step value updates are to be conservative about prices in order for revenue maximization
  - More interval updates mean interacting with more buyers to learn v
  - That is, **slower learning** but **higher revenue**

Well, This is Not the End Yet . . .

- Here, it is crucial that each buyer only shows up once
- What if the same buyer shows up repeatedly?
  - In fact, this is more realistic
  - E.g., in online advertising, buyer = an advertiser
- How should a (repeatedly showing up) buyer behave if he knows seller is learning her value v and then uses it to set a price for her?

Open Research Questions:

1. How to design pricing schemes for a repeatedly showing up buyer to maximize revenue when the buyer knows you are learning his value?
2. How to generalize to selling multiple products?

Thank You

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