CS6501: Topics in Learning and Game Theory (Spring 2021)

Introduction

Instructor: Haifeng Xu

Outline

- Course Overview
- Administrivia
- An Example

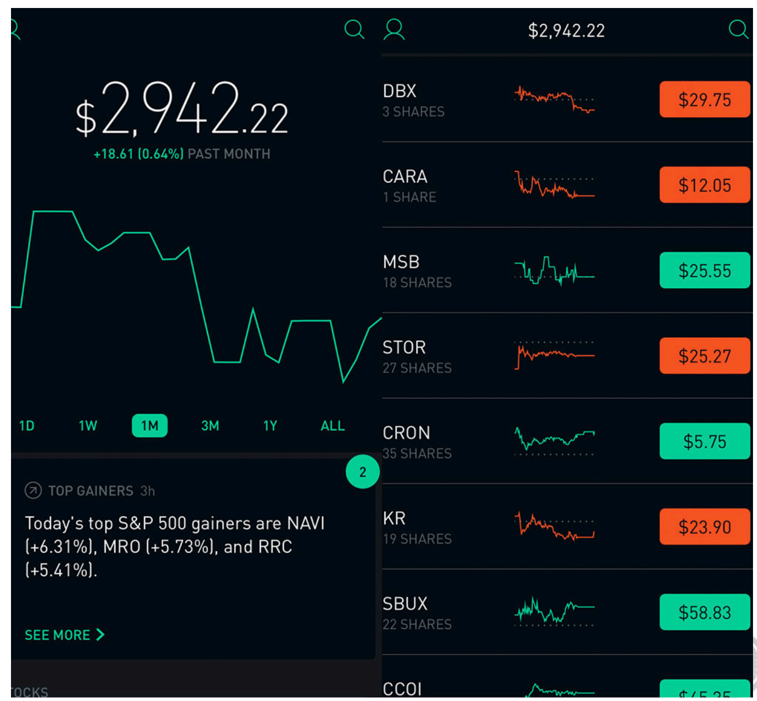
Single-Agent Decision Making

- A decision maker picks an action $x \in X$, resulting in utility $f(x)$
- Typically an optimization problem:

$$\begin{array}{ll} \text{minimize (or maximize)} & f(x) \\ \text{subject to} & x \in X \end{array}$$

  - x: decision variable
  - $f(x)$: objective function
  - X: feasible set/region
  - Optimal solution, optimal value

- Example 1: minimize x^2, s.t. $x \in [-1,1]$
- Example 2: pick a road to school
- Example 3: invest a subset of stocks



Multi-Agent Decision Making

- Usually, your payoffs affected not only by your actions, but also others'
- Agent i's utility $f_i(x_i, x_{-i})$ depends on his own action x_i, as well as other agents' actions x_{-i}
- Is this still an optimization problem? Should each agent i just pick $x_i \in X_i$ to minimize $f_i(x_i, x_{-i})$?
  - x_{-i} is not under i's control
  - Think of rock-paper-scissor game
- Examples: stock investment, routing, sales, even taking courses…

Example I: Prisoner's Dilemma

- Two members A,B of a criminal gang are arrested
- They are questioned in two separate rooms
  - No communications between them

A \ B	B stays silent	B betrays
A stays silent	-1 / -1	0 / -3
A betrays	-3 / 0	-2 / -2

Q: How should each prisoner act?

- Betray is always the best action

Example I: Prisoner's Dilemma

- Two members A,B of a criminal gang are arrested
- They are questioned in two separate rooms
  - No communications between them

B / A	B stays silent	B betrays
A stays silent	-1, -1	$\boxed{-3}$, 0
A betrays	0, -3	$\boxed{-2}$, -2

Q: How should each prisoner act?

- Betray is always the best action

Example I: Prisoner's Dilemma

- Two members A,B of a criminal gang are arrested
- They are questioned in two separate rooms
  - No communications between them

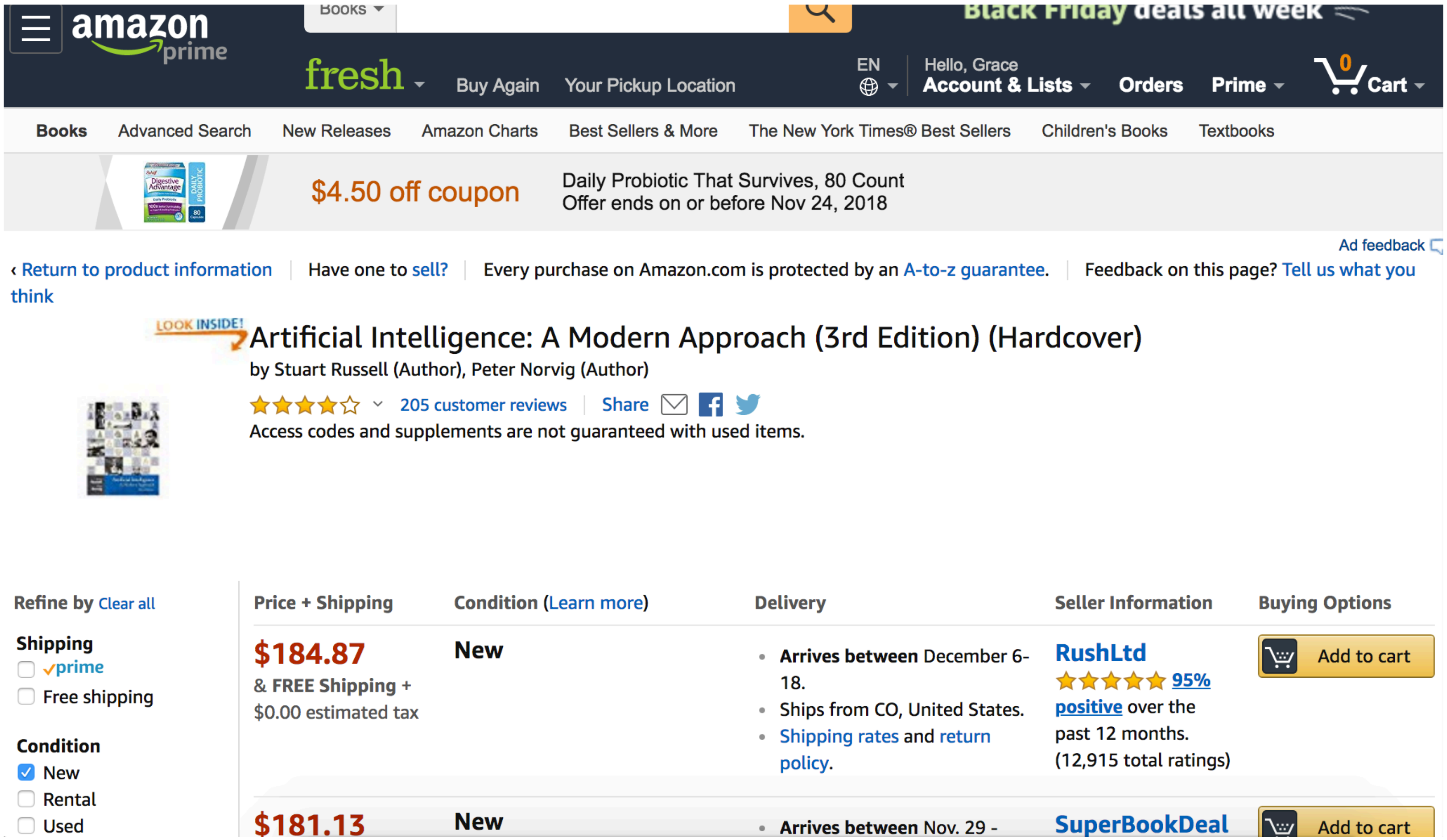
B / A	B stays silent	B betrays
A stays silent	-1 / -1	0 / -3
A betrays	-3 / 0	-2 / -2

equilibrium

Q: How should each prisoner act?

- Betray is always the best action
- But, (-1,-1) is a better outcome for both
- Why? What goes wrong?
  - Selfish behaviors lead to inefficient outcome

Example II: Markets on Amazon

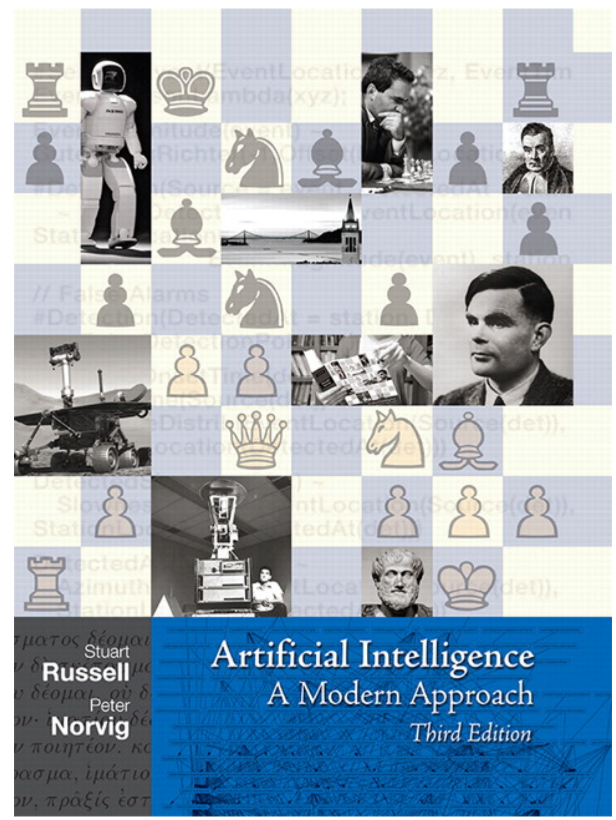


Example II: Markets on Amazon

- Assume people will buy if the book price ≤ \$200
- Product cost = \$20

If the market has only one book seller…

Q: What price should this monopoly set?

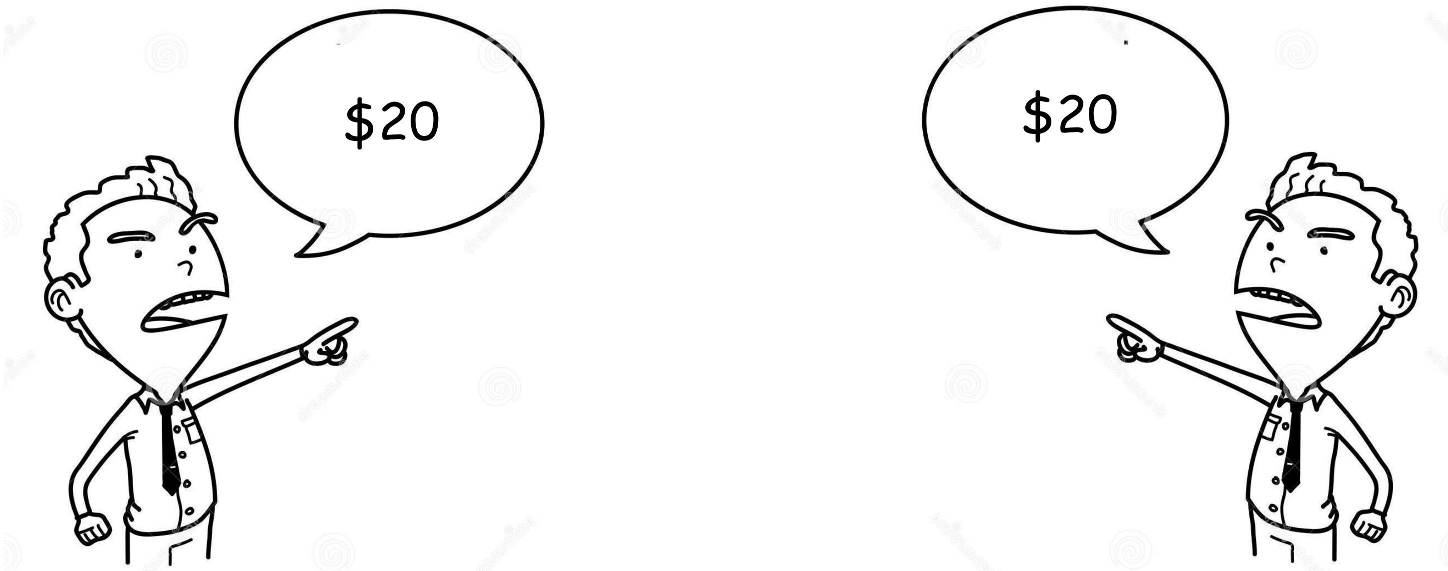


Example II: Markets on Amazon

- Assume people will buy if the book price ≤ $200
- Product cost = $20

What if the market has two book sellers…

Q: What price should each seller set?



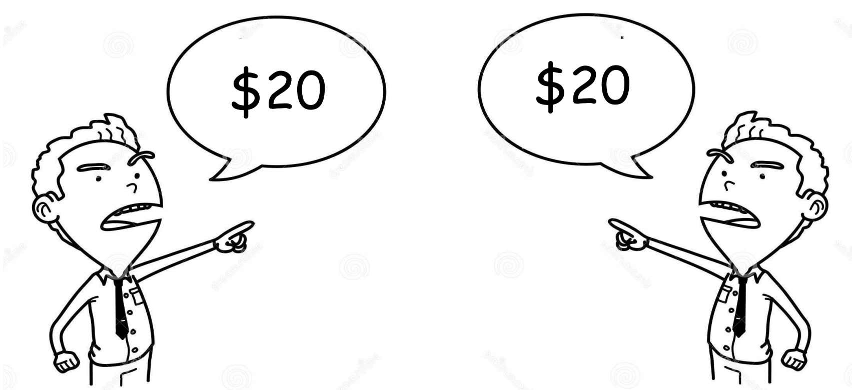
Example II: Markets on Amazon

- ➢ Assume people will buy if the book price ≤ $200
- ➢ Product cost = $20

What if the market has two book sellers…

Q: What price should each seller set?

- ➢ The market reaches a "stable status" (a.k.a., equilibrium)
- ➢ Nobody can benefit via *unilateral deviation*



- Bertrand competition
- Selfish behaviors result in inefficiency (for sellers)

Game Theory

Game Theory studies multiple-agent decision making in competitive scenarios where an agent's payoff depends on other agents' actions.

- Fundamental concept --- Equilibrium
  - A "stable status" at which any agent cannot improve his payoff through unilateral deviation
  - If exits, it should be what we expect to happen
  - Resembles "optimal decision" in single-agent case
- A central theme in game theory is to study the equilibrium
  - Different "types" of equilibria
  - May not exist; even exist, not necessarily unique
  - Understand properties of equilibrium, compute equilibria, how to improve inefficiency of equilibrium . . .

Machine Learning

- Difficult to give a universal definition
- At a high level, the task is to learn a function $f: X \rightarrow Y$, where $(\mathrm{x}, \mathrm{y}) \in X \times Y$ is drawn from some distribution D
  - Input: a set of samples $\{(x_i, y_i)\}_{i=1,2,\cdots,n}$ drawn from D
  - Output: an algorithm $A: X \rightarrow Y$ such that $A(x) \approx f(x)$ (usually measured by some loss function)
- Examples
  - Classification: $X =$ feature vectors; $Y = \{0,1\}$
  - Regression: $X =$ feature vectors; $Y = \mathbb{R}$
  - Reinforcement learning has a slightly different setup, but can be thought as $X =$ state space, Y = action space

Problems at Interface of Learning and Game Theory

- If a game is unknown or too complex, can players learn to play the game optimally?
  - Yes, sometimes – no regret learning and convergence to equilibrium
- Can game-theoretic models inspire machine learning models?
  - Yes, GANs which are zero-sum games
- Data is the fuel for ML – Can we collect high-quality data from crowd?
  - Yes, via information elicitation mechanisms
- We know how to learn to recognize faces or languages, but can we also learn the design of games to achieve some goal?
  - Yes, learning optimal auctions, product pricing schemes, etc
- Gaming/strategic behaviors in ML? How to handle them?
  - Yes, e.g, learn whether to give loans to someone or whether to admit a student to UVA based on their features
- . ..

Goodhart's Law

When a measure becomes a target, it ceases to be a good measure

Main Topics of This Course

First Half: Machine learning for game theory

- No regret learning and its convergence to equilibrium
- Learning optimal auction mechanisms

Second Half: Game theory for machine learning

- Incentivize high-quality data via information elicitation (a.k.a., crowdsourcing)
- Handle gaming behaviors in machine learning
  - Particularly, learning from strategic data sources, and fairness

Main Topics of This Course

First Half: Machine learning for game theory

- No regret learning and its convergence to equilibrium
- Learning optimal auction mechanisms

Second Half: Game theory for machine learning

- Incentivize high-quality data via information elicitation (a.k.a., crowdsourcing)
- Handle gaming behaviors in machine learning
  - Particularly, learning from strategic data sources, and fairness

Only cover fundamentals of each direction

Course Goal

- Get familiar with basics of game theory and learning
- Understand machine learning questions in game-theoretic settings, and how to deal with some of them
- Understand gaming behaviors in machine learning applications, and how to deal with some of them
- Can understand cutting-edge research papers in relevant areas

Targeted Audience of This Course

- Anyone planning to do research at the interface of game theory (or algorithm design) and machine learning
  - This is a new research direction with many opportunities/challenges
  - Recent breakthrough in no-limit poker is an example

Targeted Audience of This Course

- Anyone planning to do research at the interface of game theory (or algorithm design) and machine learning
  - This is a new research direction with many opportunities/challenges
  - Recent breakthrough in no-limit poker is an example
- Anyone interested in theoretical ML, game theory, human factors in learning, AI
  - As more and more ML systems interact with human beings, such game-theoretic reasoning becomes increasingly important
  - With more techniques developed for ML, they also broadened our toolkits for designing and solving games
- Anyone interested in understanding basics of game theory and learning

Who May not Be Suitable for This Course?

- Those who do not satisfy the prerequisites "in practice"
- Those who are looking for a recipe to implement ML/DL algorithms, or want to learn how to use TensorFlow, PyTorch, etc.
  - This is primarily a theory course
  - We will mostly focus on simple/basic yet theoretically insightful problems
  - The course is proof based – we will not write code

Outline

- Course Overview
- Administrivia
- An Example

Basic Information

- Course time: Tuesday/Thursday, 2:00 pm – 3:15 pm
- Lecture: online synchronous via Zoom
- Instructor: Haifeng Xu
  - Email: hx4ad@virginia.edu
  - Office Hour: Tue 3:15 – 4:15 pm
- TAs
  - **Jibang Wu**: office hour Mon 2 – 3 pm, Zoom
  - **Fan Yao**: office hour Friday 3 – 4 pm, Zoom
- Depending on demand, can add more office hours (let us know!)
- Couse website: http://www.haifeng-xu.com/cs6501sp21
- References: linked papers/notes on website, no official textbooks
  - Slides will be posted *after* lecture

Prerequisites

- Mathematically mature: be comfortable with proofs
- Sufficient exposures to algorithms/optimization
  - CS 6161 and equivalent, or
  - CS 4102 and you did really well
  - We will cover some basics of optimization

Requirements and Grading

- 4-5 homeworks, 60% of grade.
  - Proof based, and will be challenging
  - Discussion allowed, even encouraged, but must write up solutions independently
  - Must be written up in Latex – hand-written solutions will not be accepted
  - One late homework allowed, at most 2 days
- Research project, 40% of grade. Project instructions will be posted on website later.
  - Team up: 2 – 4 people per team
  - Can thoroughly survey a research field, or
  - Study a relevant research question, e.g., arising from your own research
  - Presentation form: a report in PDF
- FYI: no need to worry about your grade if you do invest time

If you have any suggestions/comments/concerns, feel free to email me.

Outline

- Course Overview
- Administrivia
- An Example

Learning to Sell a Product

- You are a product seller facing N unknown buyers
- These buyers all value your product at the same $v \in [0,1]$, which however is *unknown* to you
- Buyers come in sequence $1,2,\cdots,N$; For each buyer, you can choose a price p and ask him whether he is willing to buy the product
  - If $v \geq p$, she/he purchases; otherwise not

Learning to Sell a Product

- You are a product seller facing N unknown buyers
- These buyers all value your product at the same $v \in [0,1]$, which however is *unknown* to you
- Buyers come in sequence $1,2,\cdots,N$; For each buyer, you can choose a price p and ask him whether he is willing to buy the product
  - If $v \geq p$, she/he purchases; otherwise not
- How to quickly learn these buyers' value v within precision $\epsilon = \frac{1}{N}$?
  - This is a pure learning problem
  - (Well, you may directly ask a buyer's value, but guess what will happen?)
- Answer: $\log(N)$ rounds via $\mathsf{BinarySearch}$

Learning to Sell a Product

- You are a product seller facing N unknown buyers
- These buyers all value your product at the same $v \in [0,1]$, which however is *unknown* to you

Let us move to a natural game-theoretic setup ……

- You have an ultimate objective of maximizing your revenue, but do not really care about learning the v (though you may have to)
- How much revenue can BinarySearch secure?
  - May get really unlucky in first $\log(N)$ rounds and no sale happened
  - After $\log(N)$ rounds, can set a price $p \geq \tilde{v} - 1/N$ ($\tilde{v}$ is learned value)

$$\text{Rev} = \underbrace{0}_{\text{First } \log(N) \text{ rounds}} + \underbrace{(N - \log N)(v - \frac{2}{N})}_{\text{Remaining rounds}} \approx vN - v \log N \text{ -2}$$

Regret as Performance Measure

- To measure algorithm performance, we use regret

Regret $:=$ how much less is an algorithm's utility compared to the (idealized) case where we know v.

- Had we know v, should just price the product at $p = v$, earning vN
- The regret is then

$$\text{Regret(binary search)} \approx vN - [vN \ - v \log N \ -2] = v \log N + 2$$

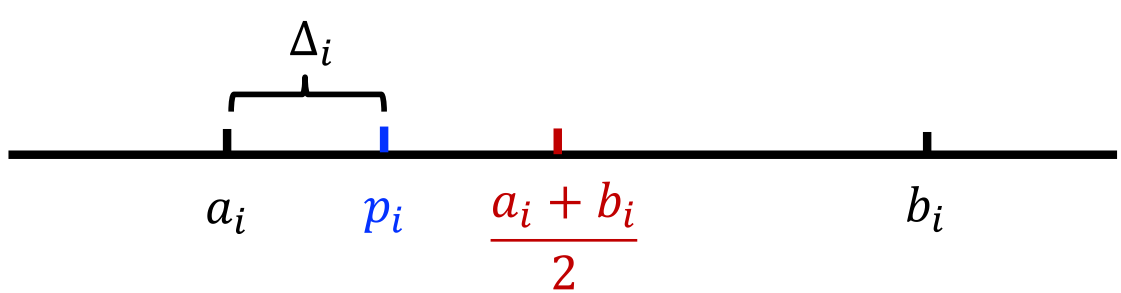
Q: Is this the best (i.e., the smallest) regret?

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2 \log \log N)$

Why BinarySearch may be bad?

- For buyer i, BinarySearch maintains an interval bound $[a_i, b_i]$ and use $p_i = (a_i + b_i)/2$ for buyer i
  - This learns v as quickly as possible
  - But maybe bad for revenue since we will get 0 revenue if $p_i > v$, and $p_i = (a_i + b_i)/2$ may be too high/aggressive
- Algorithm idea: use more conservative prices



An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2\log\log N)$

The Algorithm (note $v \in [0,1]$):

- Maintains an interval bound $[a_i, b_i]$ and a step size Δ_i
- Offer price $p_i = a_i + \Delta_i$ for buyer i

- If i accepts, update $a_{i+1} = p_i$, $b_{i+1} = b_i$, $\Delta_{i+1} = \Delta_i$
- Otherwise, update $a_{i+1} = a_i$, $b_{i+1} = p_i$, $\Delta_{i+1} = (\Delta_i)^2$
- Start with $a_1 = 0$, $b_1 = 1$, $\Delta_1 = 1/2$; Once $b_i - a_i \le \frac{1}{N}$, always use $p = a_i$ afterwards

Remark: searching smaller region with smaller step size.

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2 \log \log N)$

Algorithm analysis:

Claim 1: The step size Δ_i takes values 2^{-2^j} for $j = 0, 1, \cdots$. Moreover, whenever $\Delta_{i+1} = (\Delta_i)^2$ happens, $b_{i+1} - a_{i+1} = \sqrt{\Delta_{i+1}}$.

Proof

- Recall $\Delta_1 = \frac{1}{2} = 2^{-2^0}$, and step size update $\Delta_{i+1} = (\Delta_i)^2$
- If $\Delta_i = 2^{-2^j}$, then $(\Delta_i)^2 = 2^{-2^j - 2^j} = 2^{-2^{j+1}}$
- When $\Delta_{i+1} = (\Delta_i)^2$ happens, $b_{i+1} - a_{i+1} = \Delta_i = \sqrt{\Delta_{i+1}}$

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2 \log \log N)$

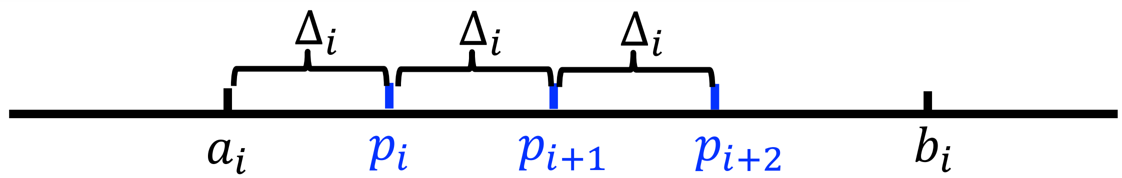
Algorithm analysis:

- After $b_i - a_i \leq \frac{1}{N}$, the total regret is at most 1
  - Because (1) regret of each step is at most $\frac{1}{N}$; (2) there are at most N rounds
- Main step is to bound regret before reaching $b_i - a_i = \frac{1}{N}$

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2\log\log N)$

Algorithm analysis:



- How many step size value updates needed to reach $b_i - a_i = \frac{1}{N}$?
  - $\log\log N$: set $2^{-2^i} = \frac{1}{N}$ → $i = \log\log N$
  - The following claim then completes the proof of the theorem

Claim 2: total regret from any step size value Δ is at most 2.

- No sale happens only once for any step size → regret at most 1
- What about the regret when sales happen?
  - Can happen at most $\sqrt{\Delta}/\Delta$ times since $b_i - a_i \leq \sqrt{\Delta}$; regret from each time is at most $b_i - a_i \leq \sqrt{\Delta}$
  - Regret from sales is at most $(\sqrt{\Delta}/\Delta)\times\sqrt{\Delta} = 1$

An Algorithm with Smaller Regret

Remarks

- $O(\log\log N)$ is also the order-wise best regret [KL, FOCS'13]
- This is an example of exploration vs exploitation
  - Exploration: want to learn v
  - Exploitation: but ultimate goal is to utilize learned v to maximize revenue
  - More in later lectures…
- BinarySearch is best for exploration, but did not balance the two
- The "optimal" algorithm uses less step value updates, but more interval updates
  - Less step value updates are to be conservative about prices in order for revenue maximization
  - More interval updates mean interacting with more buyers to learn v
  - That is, **slower learning** but **higher revenue**

Well, This is Not the End Yet . . .

- Here, it is crucial that each buyer only shows up once
- What if the same buyer shows up repeatedly?
  - In fact, this is more realistic
  - E.g., in online advertising, buyer = an advertiser
- How should a (repeatedly showing up) buyer behave if he knows seller is learning her value v and then uses it to set a price for her?

Open Research Questions:

1. How to design pricing schemes for a repeatedly showing up buyer to maximize revenue when the buyer knows you are learning his value?
2. How to generalize to selling multiple products?

Thank You

Haifeng Xu

University of Virginia

hx4ad@virginia.edu