CS6501: Topics in Learning and Game Theory (Fall 2019)

Introduction

Instructor: Haifeng Xu

Outline

- Course Overview
- Administrivia
- An Example

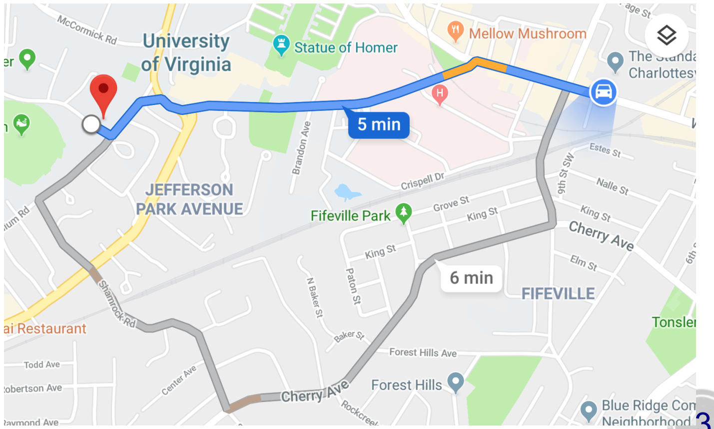
Single-Agent Decision Making

- A decision maker picks an action $x \in X$, resulting in utility $f(x)$
- Typically an optimization problem:

$$\begin{array}{ll} \text{minimize (or maximize)} & f(x) \\ \text{subject to} & x \in X \end{array}$$

  - x: decision variable
  - $f(x)$: objective function
  - X: feasible set/region
  - Optimal solution, optimal value

- Example 1: minimize x^2, s.t. $x \in [-1,1]$
- Example 2: pick a road to school



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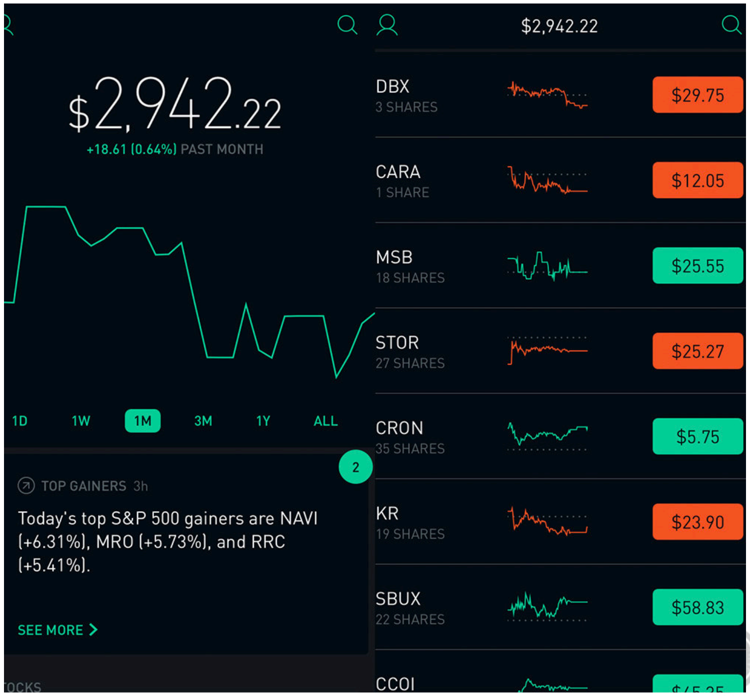


- Example 1: minimize x^2, s.t. $x \in [-1,1]$
- Example 2: pick a road to school
- Example 3: invest a subset of stocks

Multi-Agent Decision Making

- Usually, your payoffs affected not only by your actions, but also others'
- Agent i's utility $f_i(x_i, x_{-i})$ depends on his own action x_i, as well as other agents' actions x_{-i}
- Is this still an optimization problem? Should each agent i just pick $x_i \in X_i$ to minimize $f_i(x_i, x_{-i})$?
  - x_{-i} is not under i's control
  - Think of rock-paper-scissor game
- Examples: stock investment, routing, sales, even taking courses…

Example 1: Prisoner's Dilemma

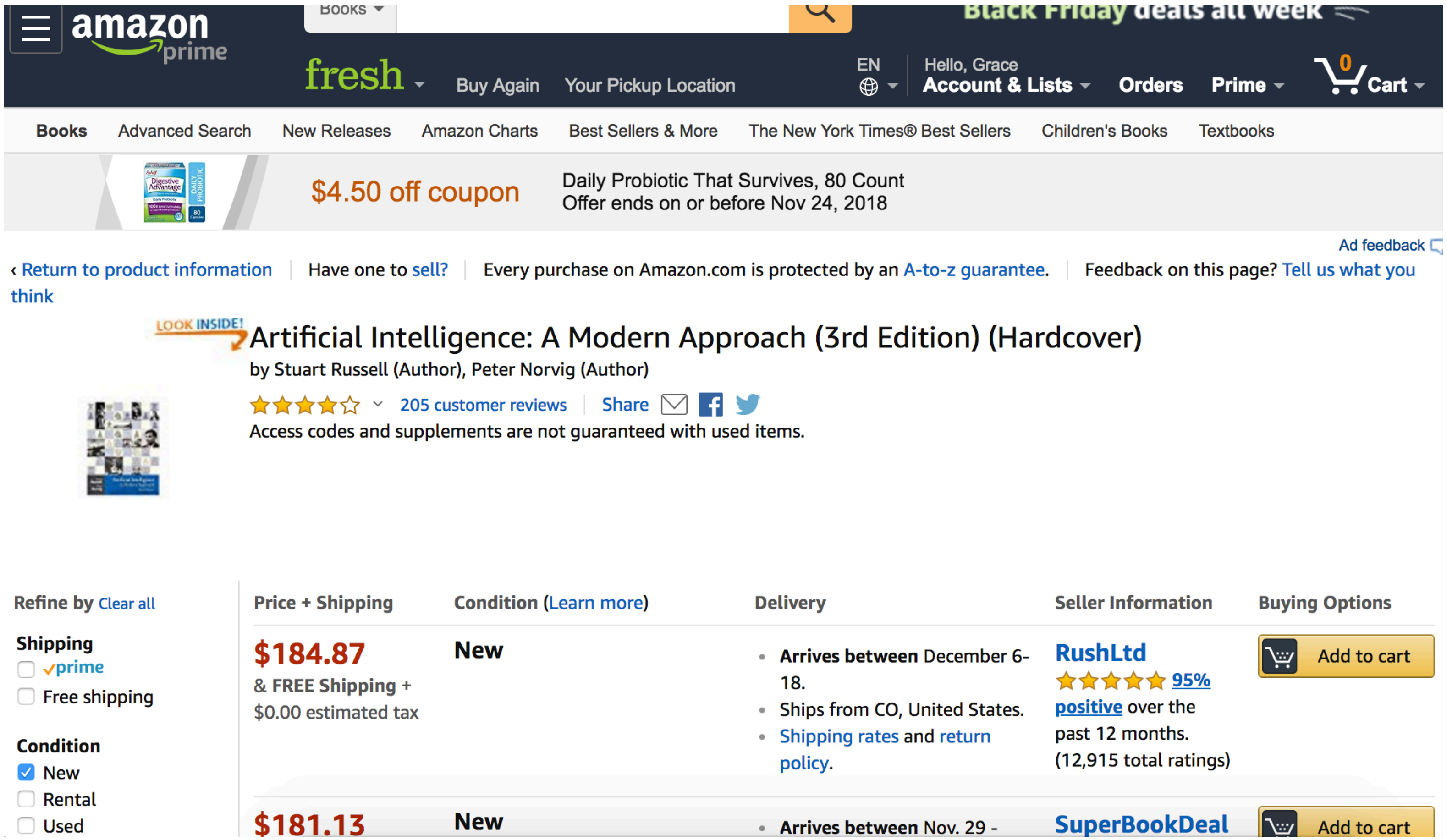
- ➢ Two members A,B of a criminal gang are arrested
- ➢ They are questioned in two separate rooms
  - ❖ No communications between them

B A	B stays silent	B betrays
A stays silent	-1 -1	0 -3
A betrays	-3 0	-2 -2

Q: How should each prisoner act?

- ➢ Both of them betray
- ➢ (-1,-1) is the best, but is not a stable status
  - • Selfish behaviors result in inefficiency

Example II: Markets on Amazon

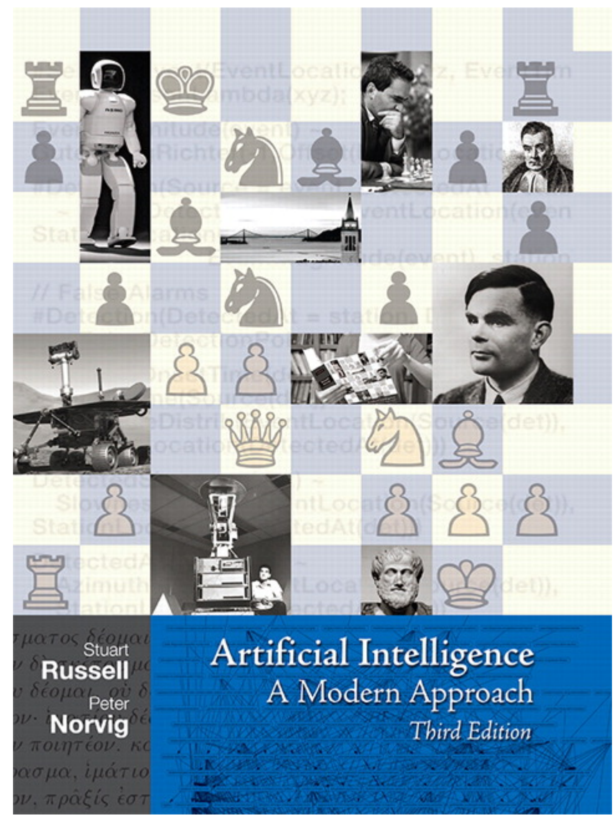


Example II: Markets on Amazon

- Assume people will buy if the book price ≤ $200
- Product cost = $20

If the market has only one book seller…

Q: What price should this monopoly set?



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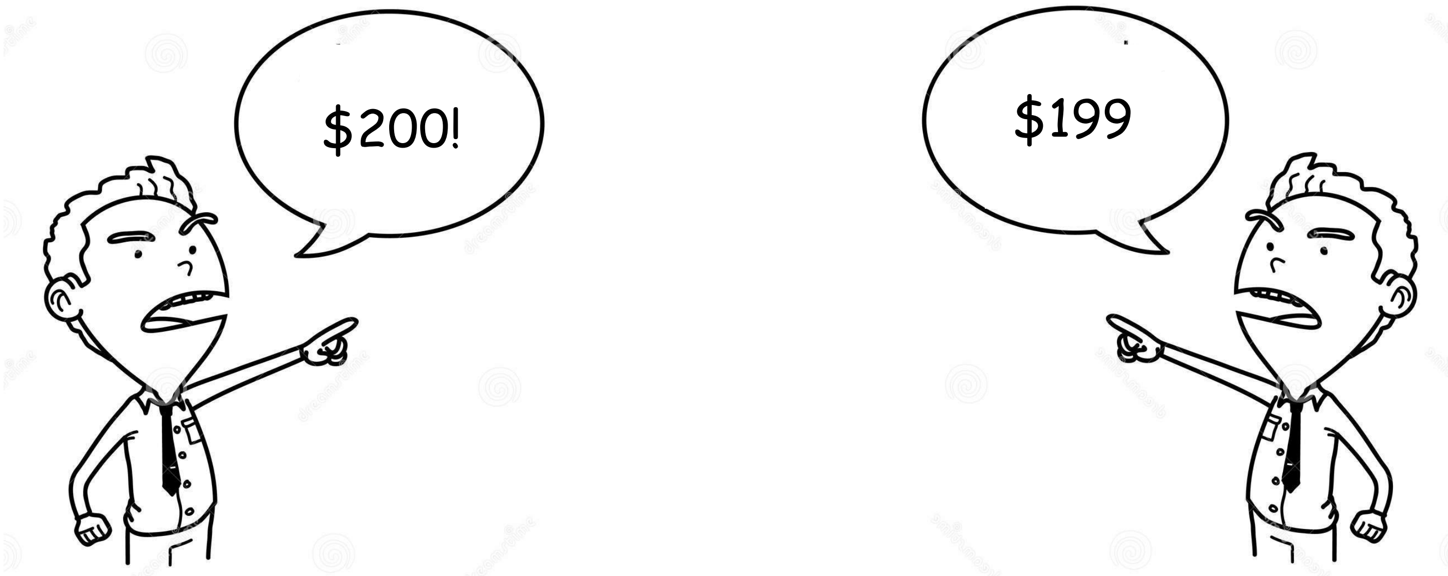
What if the market has two book sellers…

Q: What price should each seller set?



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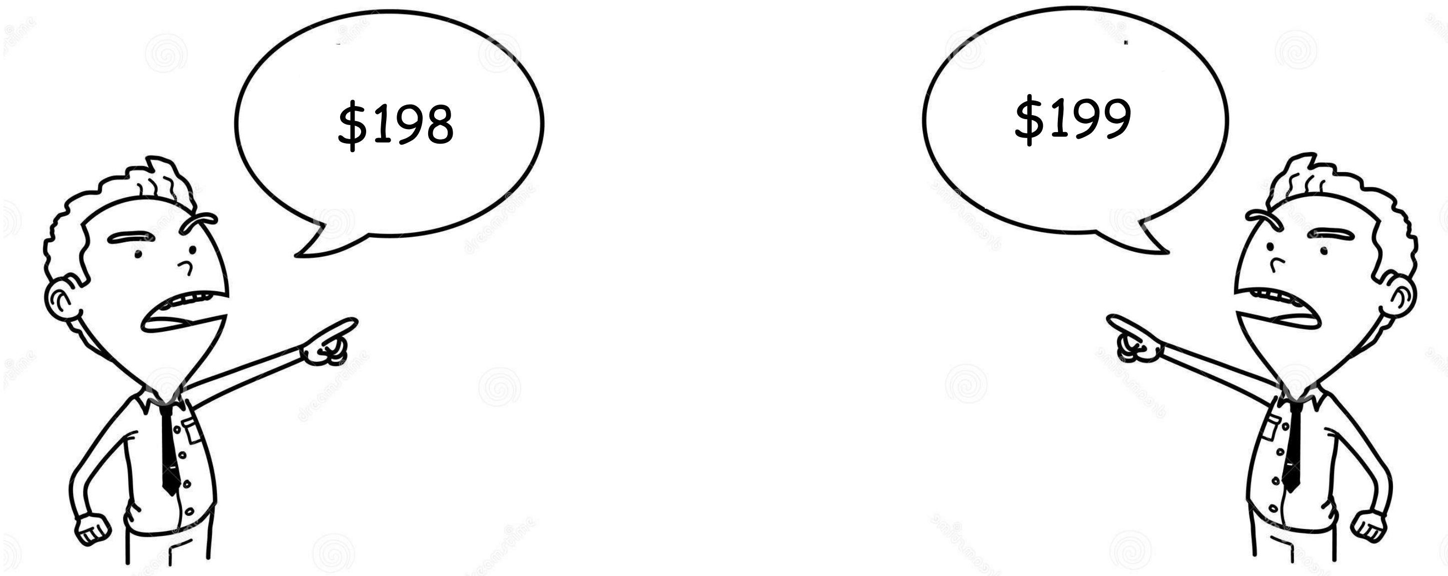

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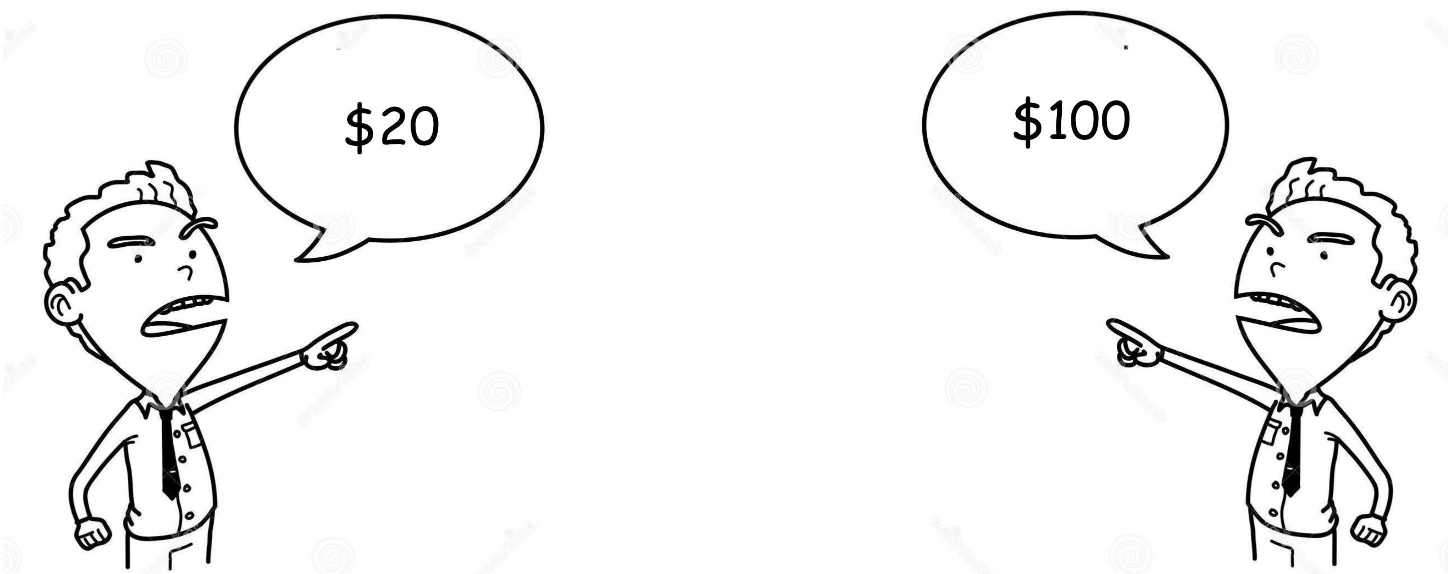

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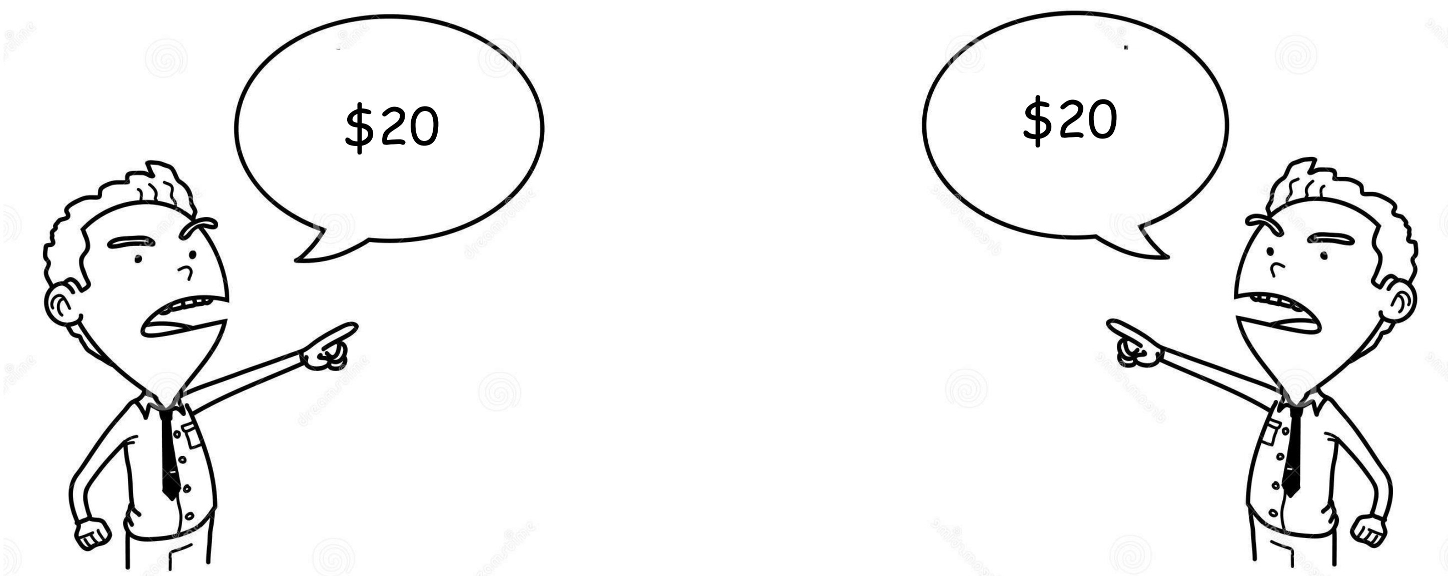

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- Assume people will buy if the book price ≤ \$200
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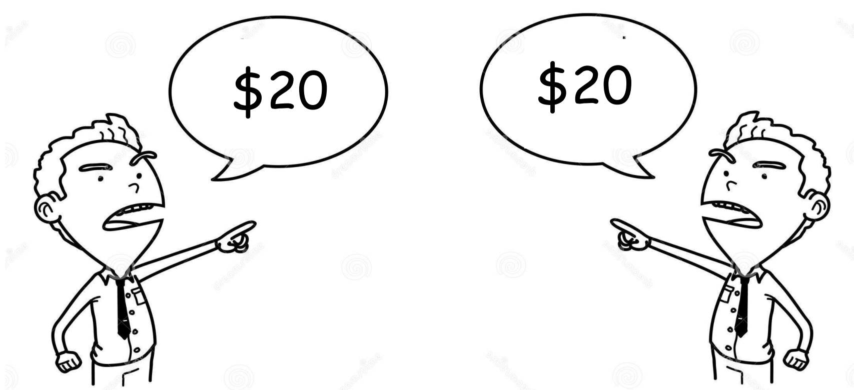
What if the market has two book sellers…

Q: What price should each seller set?

- The market reaches a "stable status" (a.k.a., equilibrium)
- Nobody can benefit via *unilateral deviation*



- Bertrand competition
- Selfish behaviors result in inefficiency (to sellers)

Game Theory

Game Theory studies multiple-agent decision making in competitive scenarios where an agent's payoff depends on other agents' actions.

- Fundamental concept --- Equilibrium
  - A "stable status" at which any agent cannot improve his payoff through unilateral deviation
  - If exits, it should be what we expect to happen
  - Resembles "optimal decision" in single-agent case
- A central theme in game theory is to study the equilibrium
  - Different definitions of equilibria
  - May not exist; even exist, not necessarily unique
  - Understand properties of equilibrium, compute equilibria, how to improve inefficiency of equilibrium . . .

Machine Learning

- Difficult to give a universal definition
- At a high level, the task is to learn a function $f: X \rightarrow Y$, where $(\mathrm{x}, \mathrm{y}) \in X \times Y$ is drawn from some distribution D
  - Input: a set of samples $\{(x_i, y_i)\}_{i=1,2,\cdots,n}$ drawn from D
  - Output: an algorithm $A: X \rightarrow Y$ such that $A(x) \approx f(x)$ (usually measured by some loss function)
- Examples
  - Classification: $X =$ feature vectors; $Y = \{0,1\}$
  - Regression: $X =$ feature vectors; $Y = \mathbb{R}$
  - Reinforcement learning has a slightly different setup, but can be thought as $X =$ state space, Y = action space

Problems at Interface of Learning and Game Theory

- If a game is unknown or too complex, can players learn to play the game optimally?
  - Yes, sometimes – no regret learning and convergence to equilibrium
- Can game-theoretic models inspire machine learning models?
  - Yes, GANs which are zero-sum games
- Data is the fuel for ML – Can we collect high-quality data from crowd?
  - Yes, via information elicitation mechanisms
- We know how to learn to recognize faces or languages, but can we also learn to design games to achieve some goal?
  - Yes, learning optimal auction mechanisms
- Game-theoretic/strategic behaviors in ML? How to handle them?
  - Yes, e.g, learn whether to give loans to someone or whether to admit a student to UVA based on their features
- . ..

Main Topics of This Course

First Half: Machine learning for game theory

- No regret learning and its convergence to equilibrium
- Learning optimal auction mechanisms

Second Half: Game theory for machine learning

- Incentivize high-quality data via information elicitation (a.k.a., crowdsourcing)
- Handle strategic behaviors in machine learning
  - Particularly, learning from strategic data sources, and fairness

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First Half: Machine learning for game theory

- No regret learning and its convergence to equilibrium
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Second Half: Game theory for machine learning

- Incentivize high-quality data via information elicitation (a.k.a., crowdsourcing)
- Handle strategic behaviors in machine learning
  - Particularly, learning from strategic data sources, and fairness

Only cover fundamentals of each direction

Course Goal

- Get familiar with basics of game theory and learning
- Understand machine learning questions in game-theoretic settings, and how to deal with some of them
- Understand strategic aspects in machine learning tasks, and how to deal with some of them
- Can understand cutting-edge research papers in relevant areas

Targeted Audience of This Course

- Anyone planning to do research at the interface of game theory (or algorithm design) and machine learning
  - This is a new research direction with many opportunities/challenges
  - Recent breakthrough in no-limit poker is an example
- Anyone interested in theoretical ML, game theory, human factors in learning, AI
  - As more and more ML systems interact with human beings, such game-theoretic reasoning becomes increasingly important
  - With more techniques developed for ML, they also broadened our toolkits for designing and solving games
- Anyone interested in understanding basics of game theory and learning

Who May not Be Suitable for This Course?

- Those who do not satisfy the prerequisites "in practice"
- Those who are looking for a recipe to implement ML/DL algorithms, or want to learn how to use TensorFlow, PyTorch, etc.
  - This is primarily a theory course
  - We will mostly focus on simple/basic yet theoretically insightful problems
  - The course is proof based – we will not write code

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Basic Information

- Course time: Tuesday/Thursday, 3:30 pm – 4:45 pm
- Lecture place: Thornton Hall E303
- Instructor: Haifeng Xu
  - Email: hx4ad@virginia.edu
  - Office: Rice Hall 522
  - Office Hour: Mon 4 – 5 pm
- TAs
  - **Minbiao Han**: office hour Thur 11 – 12 pm, Olsson Hall 001
  - **Jing Ma**: office hour Tue 11 – 12 pm, Rice Hall 442
- Depending on demand, can add more office hours (let us know!)
- Couse website: http://www.haifeng-xu.com/cs6501fa19/
- References: linked papers/notes on website, no official textbooks
  - Slides will be posted *after* lecture

Prerequisites

- Mathematically mature: be comfortable with proofs
- Sufficient exposures to algorithms/optimization
  - CS 6161 and equivalent, or
  - CS 4102 and you did really well
  - We will cover some basics of optimization

Requirements and Grading

- 3-4 homeworks, 60% of grade.
  - Proof based
  - Will be challenging
  - Discussion allowed, even encouraged, but must write up solutions independently
  - Must be written up in Latex – hand-written solutions will not be accepted
  - One late homework allowed, at most 2 days
- Research project, 40% of grade. Project instructions will be posted on website later.
  - Team up: 2 – 4 people per team
  - Can thoroughly survey a research field, or
  - Study a relevant research question, e.g., arising from your own research
  - Presentation form: a report in PDF
- FYI: should not worry about your grade if you do invest time

If you have any suggestions/comments/concerns, feel free to email me.

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Learning to Sell a Product

- You are a product seller facing N unknown buyers
- These buyers all value your product at the same $v \in [0,1]$, which however is *unknown* to you
- Buyers come in sequence $1,2,\cdots,N$; For each buyer, you can choose a price p and ask him whether he is willing to buy the product
  - If $v \geq p$, she/he purchases; otherwise not

Learning to Sell a Product

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- Buyers come in sequence $1,2, \cdots, N$; For each buyer, you can choose a price p and ask him whether he is willing to buy the product
  - If $v \geq p$, she/he purchases; otherwise not
- How to quickly learn these buyers' value v within precision $\epsilon = \frac{1}{N}$?
  - This is a pure learning problem
  - (Well, you can try to directly ask a buyer's value, guess what will happen?)
- Answer: $\log(N)$ rounds via $\mathrm{BinarySearch}$

Learning to Sell a Product

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Let us move to a natural game-theoretic setup ……

- You also have an objective of maximizing your revenue, but do not really care about learning the v (though you may have to)
- How much revenue can BinarySearch secure?
  - May get really unlucky in first $\log(N)$ rounds and no sale happened
  - After $\log(N)$ rounds, can set a price $p \geq v - 1/N$

$$\text{Rev} = \underbrace{0}_{\text{First } \log(N) \text{ rounds}} + \underbrace{(N - \log N)(v - \frac{1}{N})}_{\text{Remaining rounds}} \approx vN - v\log N \text{ -1}$$

Regret as Performance Measure

- To measure algorithm performance, we use regret

Regret := how much less is an algorithm's utility compared to the (idealized) case where we know v.

- Had we know v, should just price the product at $p = v$, earning vN
- The regret is then

$$\text{Regret(binary search)} \approx vN - [vN - v \log N - 1] = v \log N + 1$$

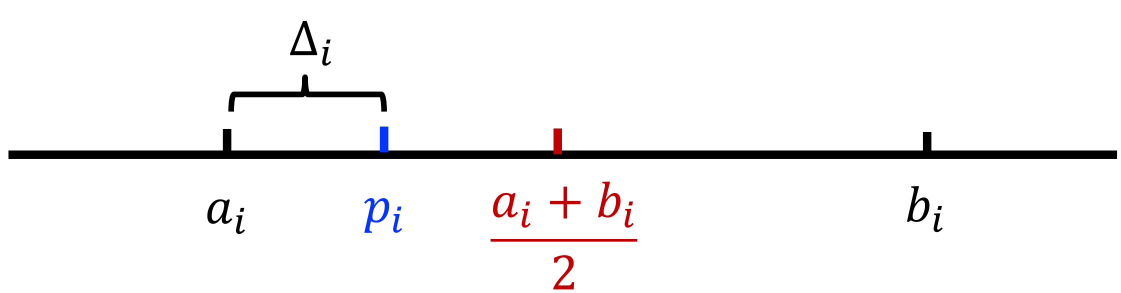
Q: Is this the best (i.e., the smallest) regret?

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2\log\log N)$

Why BinarySearch may be bad?

- For buyer i, BinarySearch maintains an interval bound $[a_i, b_i]$ and use $p_i = (a_i + b_i)/2$ for buyer i
  - This learns v as quickly as possible
  - But maybe bad for revenue since we will get 0 revenue if $p_i > v$, and $p_i = (a_i + b_i)/2$ may be too high/aggressive
- Algorithm idea: use more conservative prices



An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2\log\log N)$

The Algorithm (note $v \in [0,1]$):

- Maintains an interval bound $[a_i, b_i]$ and a step size Δ_i
- Offer price $p_i = a_i + \Delta_i$ for buyer i

- If i accepts, update $a_{i+1} = p_i$, $b_{i+1} = b_i$, $\Delta_{i+1} = \Delta_i$
- Otherwise, update $a_{i+1} = a_i$, $b_{i+1} = p_i$, $\Delta_{i+1} = (\Delta_i)^2$
- Start with $a_1 = 0$, $b_1 = 1$, $\Delta_1 = 1/2$; Once $b_i - a_i \leq \frac{1}{N}$, always use $p = a_i$ afterwards

Remark: searching smaller region with smaller step size.

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2\log\log N)$

Algorithm analysis:

Claim 1: The step size Δ_i takes values 2^{-2^j} for $j = 0, 1, \cdots$. Moreover, whenever $\Delta_{i+1} = (\Delta_i)^2$ happens, $b_{i+1} - a_{i+1} = \sqrt{\Delta_{i+1}}$.

Proof

- Recall $\Delta_1 = \frac{1}{2} = 2^{-2^0}$, and step size update $\Delta_{i+1} = (\Delta_i)^2$
- If $\Delta_i = 2^{-2^j}$, then $(\Delta_i)^2 = 2^{-2^j - 2^j} = 2^{-2^{j+1}}$

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- When $\Delta_{i+1} = (\Delta_i)^2$ happens, $b_{i+1} - a_{i+1} = \Delta_i = \sqrt{\Delta_{i+1}}$

An Algorithm with Smaller Regret

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Algorithm analysis:

- After $b_i - a_i \leq \frac{1}{N}$, the total regret is at most 1
  - Because (1) regret of each step is at most $\frac{1}{N}$; (2) there are at most N rounds
- Main step is to bound regret before reaching $b_i - a_i = \frac{1}{N}$

An Algorithm with Smaller Regret

Theorem [Kleinberg/Leighton, FOCS'03] : there is an algorithm achieving regret at most $(1 + 2 \log \log N)$

Algorithm analysis:

➢ How many step size value updates needed to reach $b_i - a_i = \frac{1}{N}$?

- $\log \log N$: set $2^{-2^i} = \frac{1}{N}$ → $i = \log \log N$
- The following claim then completes the proof of the theorem

Claim 2: total regret from any step size value Δ is at most 2.

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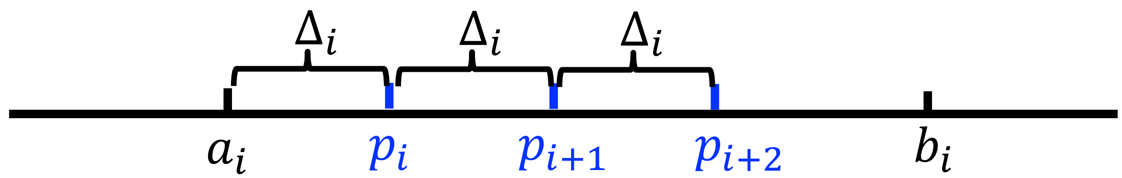

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Claim 2: total regret from any step size value Δ is at most 2.

- No sale happens only once for any step size → regret at most 1
- What about the regret when sales happen?
  - Can happen at most $\sqrt{\Delta}/\Delta$ times since $b_i - a_i \le \sqrt{\Delta}$; regret from each time is at most $b_i - a_i \le \sqrt{\Delta}$
  - Regret from sales is at most $(\sqrt{\Delta}/\Delta) \times \sqrt{\Delta} = 1$

An Algorithm with Smaller Regret

Remarks

- $O(\log\log N)$ is also the order-wise best regret [KL, FOCS'13]
- This is an example of exploration vs exploitation
  - Exploration: want to learn v
  - Exploitation: but ultimate goal is to utilize learned v to maximize revenue
  - More in later lectures…
- BinarySearch is best for exploration, but did not balance the two

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- BinarySearch is best for exploration, but did not balance the two
- The "optimal" algorithm uses less step value updates, but more interval updates
  - Less step value updates are to be conservative about prices in order for revenue maximization
  - More interval updates mean interacting with more buyers to learn v
  - That is, **slower learning** but **higher revenue**

Well, This is Not the End Yet . . .

- Here, it is crucial that each buyer only shows up once
- What if the same buyer shows up repeatedly?
  - In fact, this is more realistic
  - E.g., in online advertising, buyer = an advertiser
- How should a (repeatedly showing up) buyer behave if he knows seller is learning her value v and then uses it to set a price for her?

Open Research Questions:

1. How to design pricing schemes for a repeatedly showing up buyer to maximize revenue when the buyer knows you are learning his value?
2. How to generalize to selling multiple products?

Thank You

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