

Announcements

- HW 2 is due next Tuesday
 - No class on next Tuesday, but TAs will be here to collect HW

CS6501: Topics in Learning and Game Theory (Fall 2019)

Prediction Markets (as a Forecasting Tool)

Instructor: Haifeng Xu

JOURNAL ARTICLE

Orange Juice and Weather

Richard Roll






The American Economic Review

Vol. 74, No. 5 (Dec., 1984),
pp. 861-880 (20 pages)

Futures of orange juice can be used to predict weather



SAT	SUN	MON
		
Mostly Sunny	T-storms	Sunny
High: 76° Low: 52°	High: 72° Low: 37°	High: 55° Low: 32°

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Events of Interest for Prediction

- Will there be a HW4 for this course?
- Will UVA win NCAA championship in 2020?
- Will bit coin price exceed \$9K tomorrow?
- Will Tesla's stock exceed \$300 by the end of this year?
- Will the number of iPhones sold in 2019 exceed 150 million?
- Will Trump win the election in 2020
- Will there be a cure for cancer by 2025?
- Will the world be peaceful in 2050?
- . . .

The Prediction Problem

- An uncertain event to be predicted
 - Will Tesla stock exceed \$300 by Dec 2019?
- Dispersed information/evidence
 - Tesla employees, Tesla drivers, other EV company employees, government policy makers, etc.
- Goal: generate a prediction that is based on information from all sources
 - ML can also do prediction, but will see why markets have advantages

Bet \approx Credible Opinion

Q: will P vs NP problem be solved by the end of 20th century?



Michael Sipser

P vs NP would be solved by the end of the 20th century, if not sooner. The terms: **one ounce of pure gold**

- Other examples: stock trading, gambling, . . .
- Betting intermediaries: Wall Street, Las Vegas, InTrade, . . .

Prediction Markets

A prediction market is a **financial market** that is designed for **event prediction** via information aggregation

- Payoffs of the traded **contract** are determined by outcomes of future events








$\left\{ \begin{array}{l} \$1 \text{ if UVA wins NCAA} \\ \$0 \text{ otherwise} \end{array} \right.$

A contract

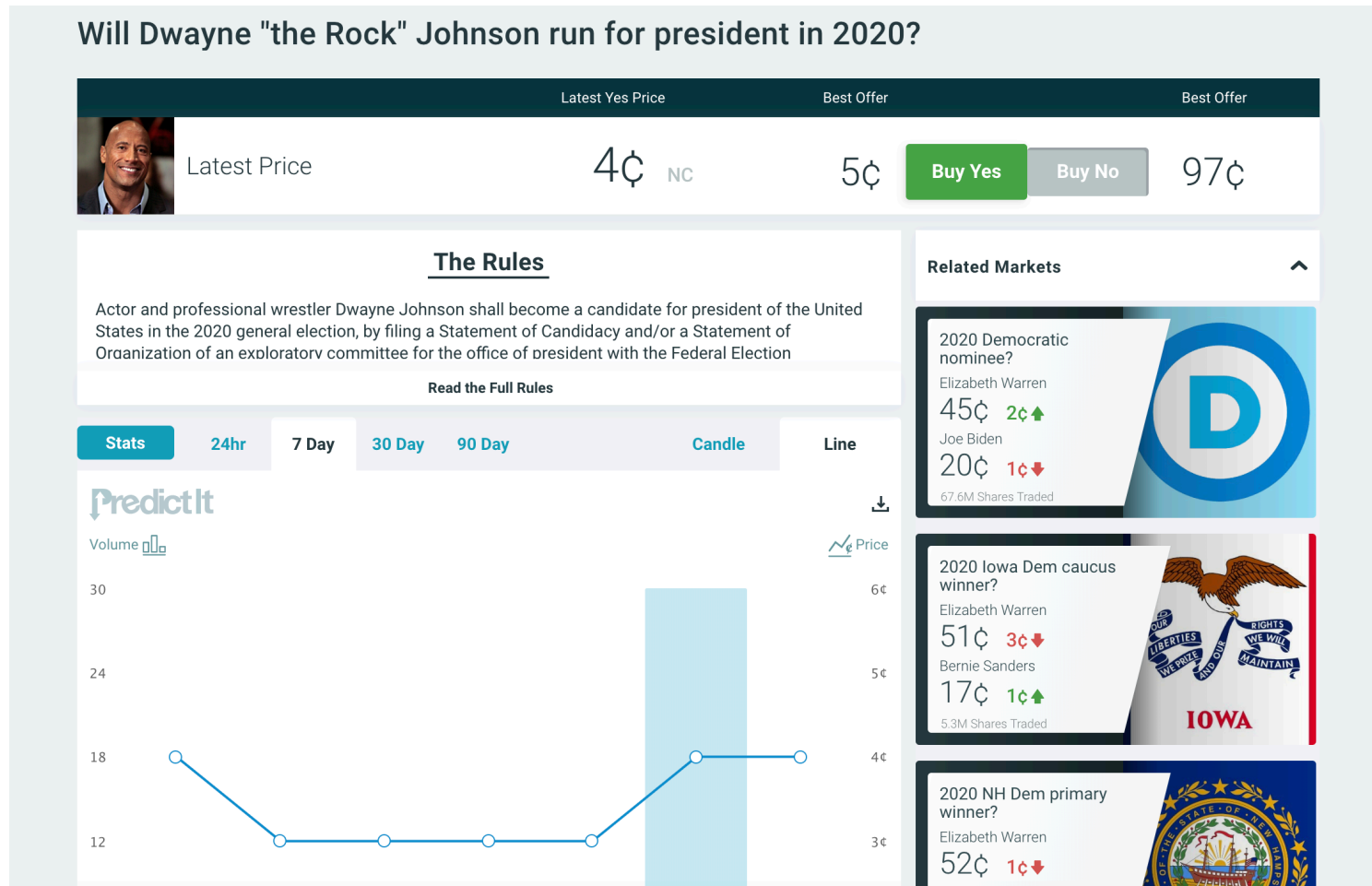
Price of a contract? $\$1 \times \text{percentage of shares that bet on UVA winning?}$

This is what we will be designing!


Prediction Markets: Examples

<div>PredictIt</div> <div>Markets</div> <div>Support</div> <div>Insights</div> <div>Leaderboard</div> <div>Login</div> <div>Sign Up</div> <div></div>						
<div>U.S. Elections</div> <div>Trump Admin</div> <div>Congress</div> <div>Justice</div> <div>World</div>						
Who will win the 2020 U.S. presidential election?						
Contract	Latest Yes Price	Best Offer				
 Donald Trump	41¢ NC	41¢	Buy Yes	Buy No	60¢	
 Elizabeth Warren	31¢ NC	32¢	Buy Yes	Buy No	69¢	
 Joe Biden	13¢ NC	13¢	Buy Yes	Buy No	88¢	
 Bernie Sanders	11¢ 2¢↑	11¢	Buy Yes	Buy No	91¢	
 Andrew Yang	8¢ NC	9¢	Buy Yes	Buy No	92¢	
 Pete Buttigieg	7¢ NC	7¢	Buy Yes	Buy No	94¢	
 Mike Pence	3¢ NC	3¢	Buy Yes	Buy No	98¢	

Prediction Markets: Examples





Prediction Markets: Examples




HI HAIFENG XU ▾ CLAIMS ▾ ABOUT BLOG
 SEARCH



SURVEYS



☐ * Demographics and Interests (Required)
 ☐ Forecasting Replications Quiz
 ☐ Statistics Background & Quiz



FORECAST TOPICS



 All Claims
  American Journal of Political Science



 Economics
  Experimental Economics

 Journal of Experimental Social Psychology
  Journal of Labor Economics

 Journal of Marketing
  Journal of Organizational Behavior

 Journal of Personality and Social Psychology
  Journal of the Academy of Marketing Science

 Management & Marketing & others
  Political Science

 Psychological Science
  Psychology

LEADERBOARDS

Expected Points ▾


1	sattuma	1,020
2	adamlgreen	1,009
3	BradleyJBaker	949
4	Luthor113	929
5	ncerutti	920
6	ulrich	871
7	stanlaurel	817
8	unipedal	803
9	ted	803
10	Achim	791
11	cteno4	782

RECENT ACTIVITY


Participants ▾

NO RESULTS FOUND.


OUR TEAM



David Pfeiffer,
Massachusetts Institute of Technology




Anna Dreber,
Stockholm School of Economics




Magnus Johannesson,
Stockholm School of Economics


Incentives, Scoring, IRB



Jing Chen,
Harvard University



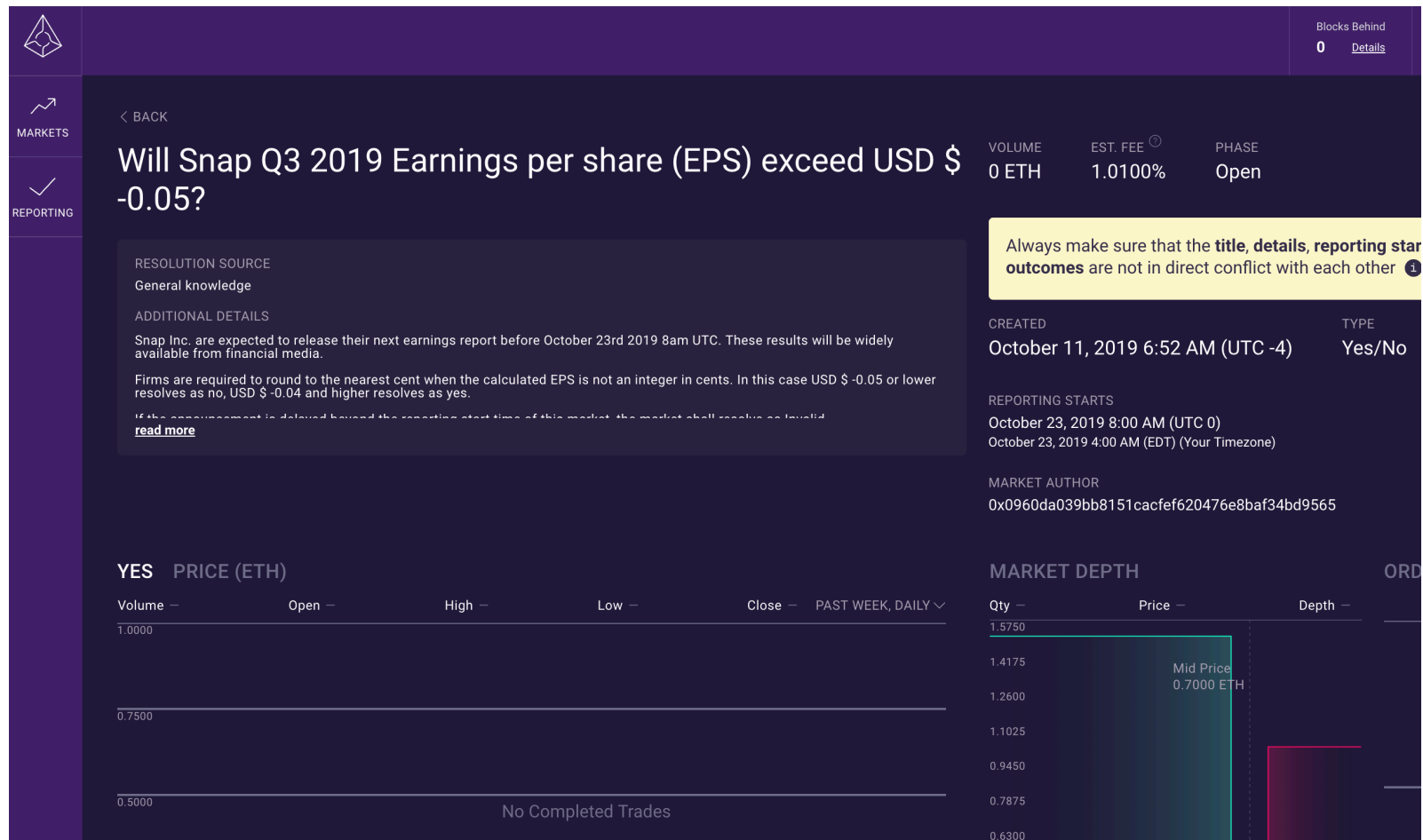
Prof. Yang Liu,
University of California Santa Cruz



Prof. Haifeng Xu,
University of Virginia

Replication Market

Prediction Markets: Examples



Augur: the first decentralized prediction markets

Does It Work?

- Yes, evidence from real markets, lab experiments, and theory
 - I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995][Rietz 1998][Berg 2001][Pennock 2002]
 - HP market beats sales forecast 6/8 [Plott 2000]
 - Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC'03][Schmidt 2002]
 - Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC'01]
 - Theory: “rational expectations” [Grossman 1981][Lucas 1972]
 - More ...

Why Can Markets Aggregate Information?

➤ Price $\approx \text{Prob}(\text{event} | \text{all information})$

\$1 if UVA wins NCAA title, \$0 otherwise

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Value of contract

?

Payoff

\$1

\$0

Event Outcome

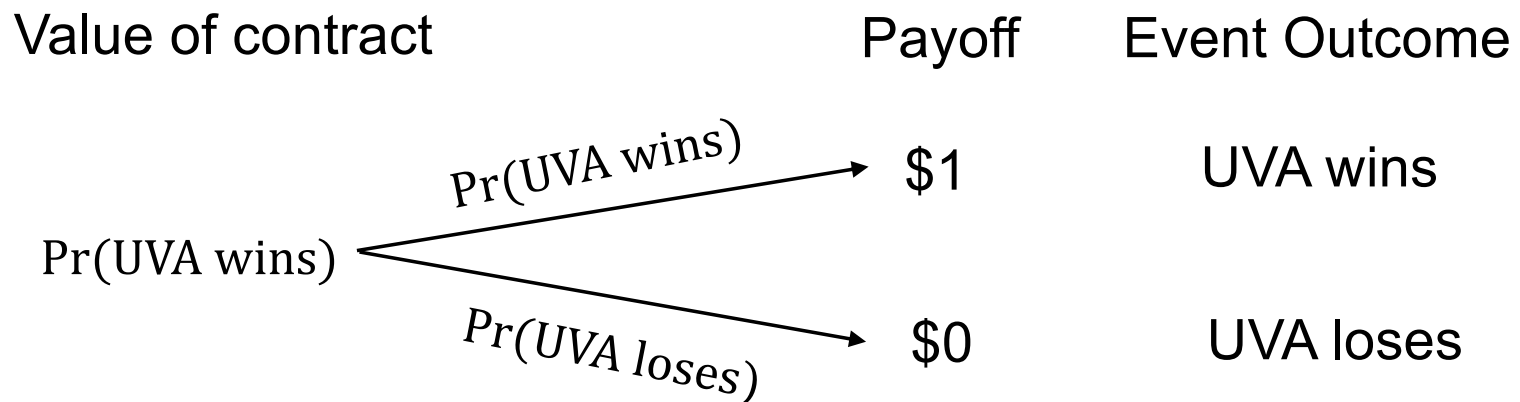
UVA wins

UVA loses

Why Can Markets Aggregate Information?

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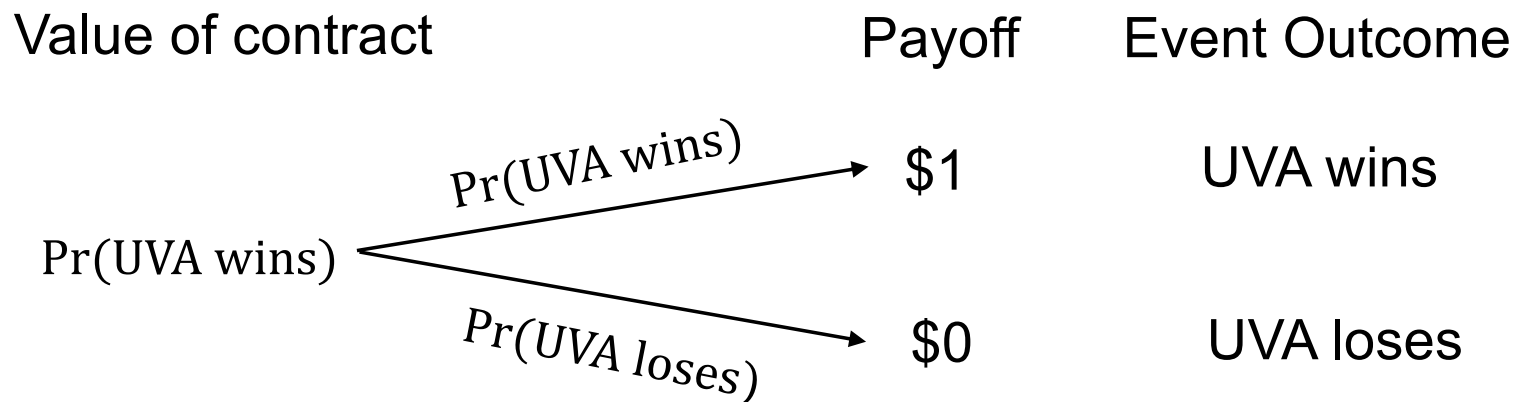
\$1 if UVA wins NCAA title, \$0 otherwise



Why Can Markets Aggregate Information?

➤ Price $\approx \text{Prob}(\text{event} | \text{all information})$

\$1 if UVA wins NCAA title, \$0 otherwise



Value of contract $\approx P(\text{UVA wins}) \approx \text{Equilibrium price}$

Market Efficiency (a design goal)

Markets vs Other Prediction Approaches

Opinion Poll

- Sampling
- No incentive to be truthful
- Equally weighted information
- Hard to be real-time

Ask Experts

- Identifying experts can be hard
- Combining opinions is difficult

Prediction Markets

- Self-selection
- Monetary incentive and more
- Money-weighted information
- Real-time
- Self-organizing

Other Prediction Approaches vs Markets

Machine Learning

- Historical data
- Assume past and future are related
- Hard to incorporate recent new information

Prediction Markets

- No need for data
- No assumption on past and future
- Immediately incorporate new information

Caveat: markets have their own problems too – manipulations, irrational traders, etc.

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets (PMs)
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Some Design Objectives of PMs

Liquidity: people can find counterparties to trade whenever they want

Bounded loss: total loss of the market institution is bounded

Market efficiency: Price reflects predicted probabilities.

Computational efficiency: The process of operating the market should be computationally manageable.

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

➤ Seller orders

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.12

➤ Seller orders

\$0.30

Continuous Double Auction (CDA) Market

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\$0.12

\$0.09

➤ Seller orders

\$0.30

\$0.17

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Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

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\$0.09

\$0.15

➤ Seller orders

\$0.17

\$0.30

\$0.13

Continuous Double Auction (CDA) Market

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➤ Seller orders

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\$0.17

\$0.30

Continuous Double Auction (CDA) Market

\$1 if UVA wins NCAA title, \$0 otherwise

➤ Buyer orders

\$0.15

\$0.12

\$0.09

Price = \$0.14

➤ Seller orders

\$0.13

\$0.17

\$0.30

What's Wrong with CDA?

➤ Thin market problem

- When there are not enough traders, trade may not happen.

➤ No trade theorem [\[Milgrom & Stokey 1982\]](#)

- Why trade? These markets are zero-sum games (negative sum w/ transaction fees)
- For all money earned, there is an equal (greater) amount lost; am I smarter than average?
- Rational risk-neutral traders will *never* trade
- But trade still happens ...

An Alternative: Market Maker (MM)



- A market maker is the market institution who sets the prices and is willing to accept orders (buy or sell) at the price specified.
- Why? **Liquidity!**
- Market makers bear risk. Thus, we desire mechanisms that can **bound the loss of market makers.**

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ An (automated) market maker (MM)

➤ Sell or buy back contracts \$1 iff e_1 $\cdot \cdot \cdot$ \$1 iff e_n

➤ Value function ($q = (q_1, \dots, q_n)$ is current sales quantity)

$$V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$$

Parameter b
adjusts liquidity

➤ Price function

$$p_i(q) = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}} = \frac{\partial V(q)}{\partial q_i}$$

➤ To buy $x \in \mathbb{R}^n$ amount, a buyer pays: $V(q + x) - V(q)$

- Negative x_i 's mean selling contracts to MM
- Negative payment means market maker pays the buyer
- Market starts with $V(0) = b \log n$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

➤ Say, you buy $x \in \mathbb{R}^n$ amount

➤ You pay $V(q + x) - V(q)$; Your expected return is $\sum_{j \in [n]} \lambda_j \cdot x_j$

➤ Expected utility is

$$U(x) = \sum_{j \in [n]} \lambda_j \cdot x_j - b \log \sum_{j \in [n]} e^{(q_j + x_j)/b} + V(q)$$

➤ Which x maximizes your utility?

$$\frac{\partial U(x)}{\partial x_i} = \lambda_i - \frac{e^{(q_i + x_i)/b}}{\sum_{j \in [n]} e^{(q_j + x_j)/b}} = 0$$

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The market price of contract i
after your purchase

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

- Why non-negative?
- Buy 0 amount leads to 0, so optimal amount is at least as good

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event e_1, \dots, e_n is $\lambda = (\lambda_1, \dots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ . Your expected utility of purchasing this amount is always non-negative.

- This is the expected utility you believe, but **may be incorrect since your λ may be inaccurate!**
- So, buy only when your prediction is really more accurate than the current market prediction
 - **Achieves market efficiency:** price = current best market prediction

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q2: If market ends up with $q = (q_1, \dots, q_n)$ shares for each contract, how much money did the MM collect?

- Answer: $V(q) - V(0) = V(q) - b \log n$
- But after event outcome is realized, MM need to pay based on contracts – what is the worst-case loss of MM?

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

➤ Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Fact. After event outcome realizes and MM pays the contract, worst case MM loses is $b \log n$ (i.e., bounded).

Proof

➤ Only one event will be realized, say it is event e_i

➤ MM utility is $V(q) - b \log n - q_i$

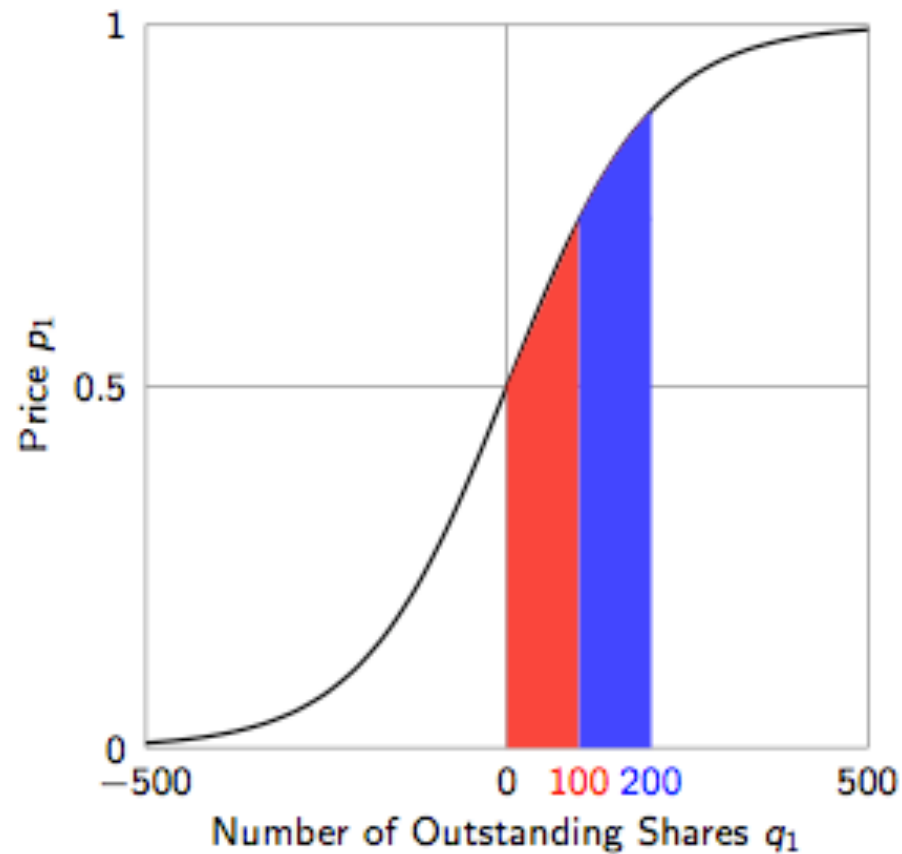
$$\geq b \log e^{q_i/b} - b \log n - q_i$$

$$\geq q_i - b \log n - q_i$$

$$\geq -b \log n$$

“=” can be achieved by letting $q_i \rightarrow \infty$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])



Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Has been implemented by several prediction markets
 - E.g., InklingMarkets, Washington Stock Exchange, BizPredict, Net Exchange, and (reportedly) at YooNew.

SELECTED PREDICTION

Barack Obama

CURRENT PRICE

\$57.02

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

Do you think:

- Chances are higher than 57.02% this will occur
- Chances are lower than 57.02% this will occur

TIP: A price of \$57.02 means there is currently a 57.0% chance this will occur.

If you think the current odds of 57% are:

☐ Way too low...

☐ Low...

☐ Just below...

☐ Advanced...

Buy 50 shares	Buy 20 shares	Buy 5 shares	Buy <input type="text"/> shares
your cost \$2,971.95	your cost \$1,159.83	your cost \$286.30	your cost ...
estimated new price \$61.84	estimated new price \$58.97	estimated new price \$57.51	estimated new price ...

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
 - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates (EWU)

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$



Action 1, $w_t(1)$



Action 2, $w_t(2)$

...



Action n , $w_t(n)$

$$\begin{aligned} w_{t+1}(i) &= w_t(i) \cdot e^{-\epsilon c_t(i)} \\ &= [w_{t-1}(i) \cdot e^{-\epsilon c_{t-1}(i)}] \cdot e^{-\epsilon c_t(i)} \\ &= \dots = e^{-\epsilon C_t(i)} \text{ where } C_t(i) = \sum_{\tau \leq t} c_\tau(i) \end{aligned}$$

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \dots, w_t(n)$
- Observe cost vector c_t , and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$
- At round $t + 1$, select action i with probability

$$\frac{w_t(i)}{W_t} = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

where $C_t = \sum_{\tau \leq t} c_\tau$ is the accumulated cost vector

This looks very much like the price function in LMSR (q is the accumulated sales quantity)

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

EWU vs LMSR

➤ Exponential Weight Update

- n actions
- Maintain weight $w_t(i)$
- Total cost $C_T(i) = \sum_{t \leq T} c_t(i)$
- Select i with prob

$$p_i = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

- Weights reflect how good an action is
- Care about worst case regret

$$C_T(\text{Alg}) - \min_i C_T(i)$$

➤ LMSR

- n contracts (i.e., outcomes)
- Maintain prices $p(i)$
- Total shares sold $q(i)$
- Price of contract i

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

- Prices reflect how probable is an event
- Care about worst case MM loss

$$(\$ \text{ received}) - \min_i q(i)$$

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market makers and no-regret learning algorithms (see [[Chen and Vaughan 2010](#)])
- Markets can potentially be a very effective forecasting tool
 - Big on-going project: “replication market” for DARPA SCORE program



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Systematizing Confidence in Open Research and Evidence (SCORE)

[Dr. Adam Russell](#)

The Department of Defense (DoD) often leverages social and behavioral science (SBS) research to design plans, guide investments, assess outcomes, and build models of human social systems and behaviors as they relate to national security challenges in the human domain. However, a number of recent empirical studies and meta-analyses have revealed that many SBS results vary dramatically in terms of their ability to be independently reproduced or replicated, which could have real-world implications for DoD's plans, decisions, and models. To help address this situation, DARPA's Systematizing Confidence in Open Research and Evidence (SCORE) program aims to develop and deploy automated tools to assign "confidence scores" to different SBS research results and claims. Confidence scores are quantitative measures that should enable a DoD consumer of SBS research to understand the degree to which a particular claim or result is likely to be reproducible or replicable. These tools will assign explainable confidence scores with a reliability that is equal to, or better than, the best current human expert methods. If successful, SCORE will enable DoD personnel to quickly calibrate the level of confidence they should have in the reproducibility and replicability of a given SBS result or claim, and thereby

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market makers and no-regret learning algorithms (see [[Chen and Vaughan 2010](#)])
- Markets can potentially be a very effective forecasting tool
 - Big on-going project: “replication market” for DARPA SCORE program
- Mechanism design for prediction tasks
 - ML is one way but not the only way of making predictions
 - But markets and ML may augment each other

Thank You

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