Announcements

- HW 2 is due next Tuesday
  - No class on next Tuesday, but TAs will be here to collect HW

CS6501: Topics in Learning and Game Theory (Fall 2019)

Prediction Markets (as a Forecasting Tool)

Instructor: Haifeng Xu

Slides of this lecture are adapted from slides by Yiling Chen

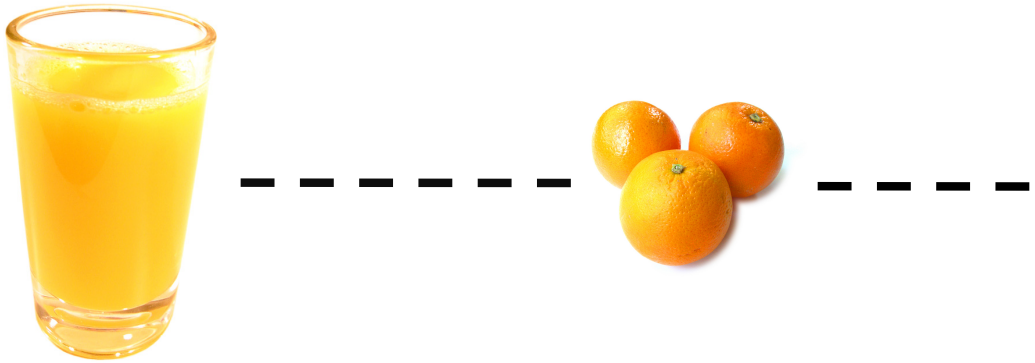
JOURNAL ARTICLE

Orange Juice and Weather

Richard Roll

The American Economic Review
Vol. 74, No. 5 (Dec., 1984), pp. 861-880 (20 pages)

Futures of orange juice can be used to predict weather



SAT	SUN	MON
Mostly Sunny	T-storms	Sunny
High: 76°	High: 72°	High: 55°
Low: 52°	Low: 37°	Low: 32°

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
  - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Events of Interest for Prediction

- Will there be a HW4 for this course?
- Will UVA win NCAA championship in 2020?
- Will bit coin price exceed $9K tomorrow?
- Will Tesla's stock exceed $300 by the end of this year?
- Will the number of iPhones sold in 2019 exceed 150 million?
- Will Trump win the election in 2020
- Will there be a cure for cancer by 2025?
- Will the world be peaceful in 2050?
- . . .

The Prediction Problem

- An uncertain event to be predicted
  - Will Tesla stock exceed $300 by Dec 2019?
- Dispersed information/evidence
  - Tesla employees, Tesla drivers, other EV company employees, government policy makers, etc.
- Goal: generate a prediction that is based on information from all sources
  - ML can also do prediction, but will see why markets have advantages

Bet ≈ Credible Opinion

Q: will P vs NP problem by solved by the end of 20'th century?

P vs NP would be solved by the end of the 20th century, if not sooner. The terms: one ounce of pure gold

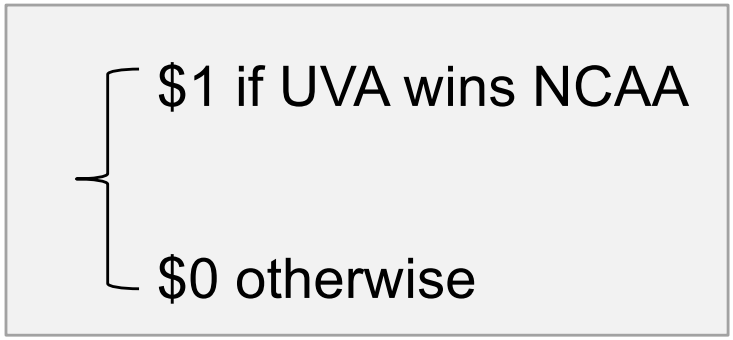
Michael Sipser

- Other examples: stock trading, gambling, . . .
- Betting intermediaries: Wall Street, Las Vegas, InTrade, . . .

Prediction Markets

A prediction market is a financial market that is designed for event prediction via information aggregation

- Payoffs of the traded contract are determined by outcomes of future events

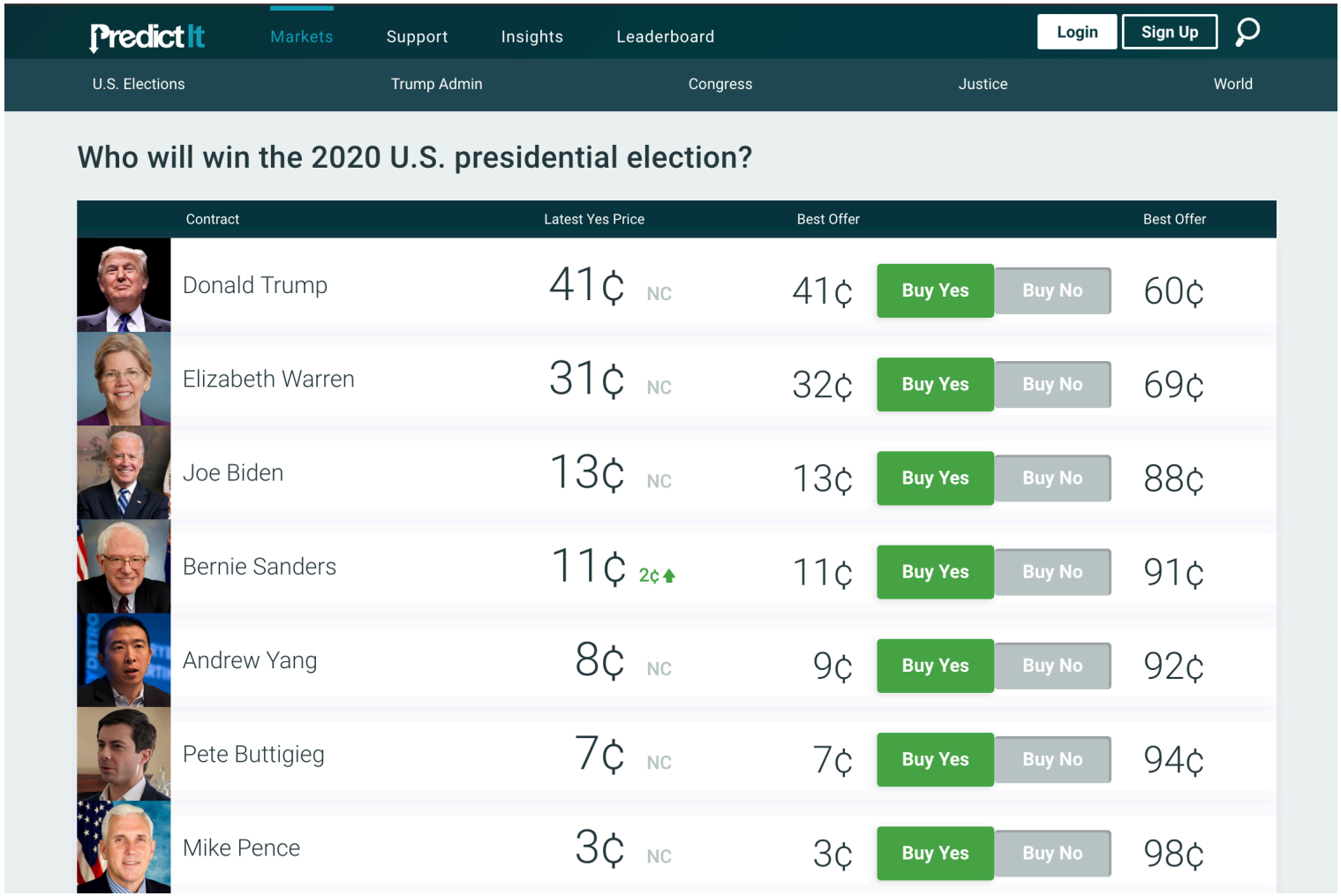


A contract

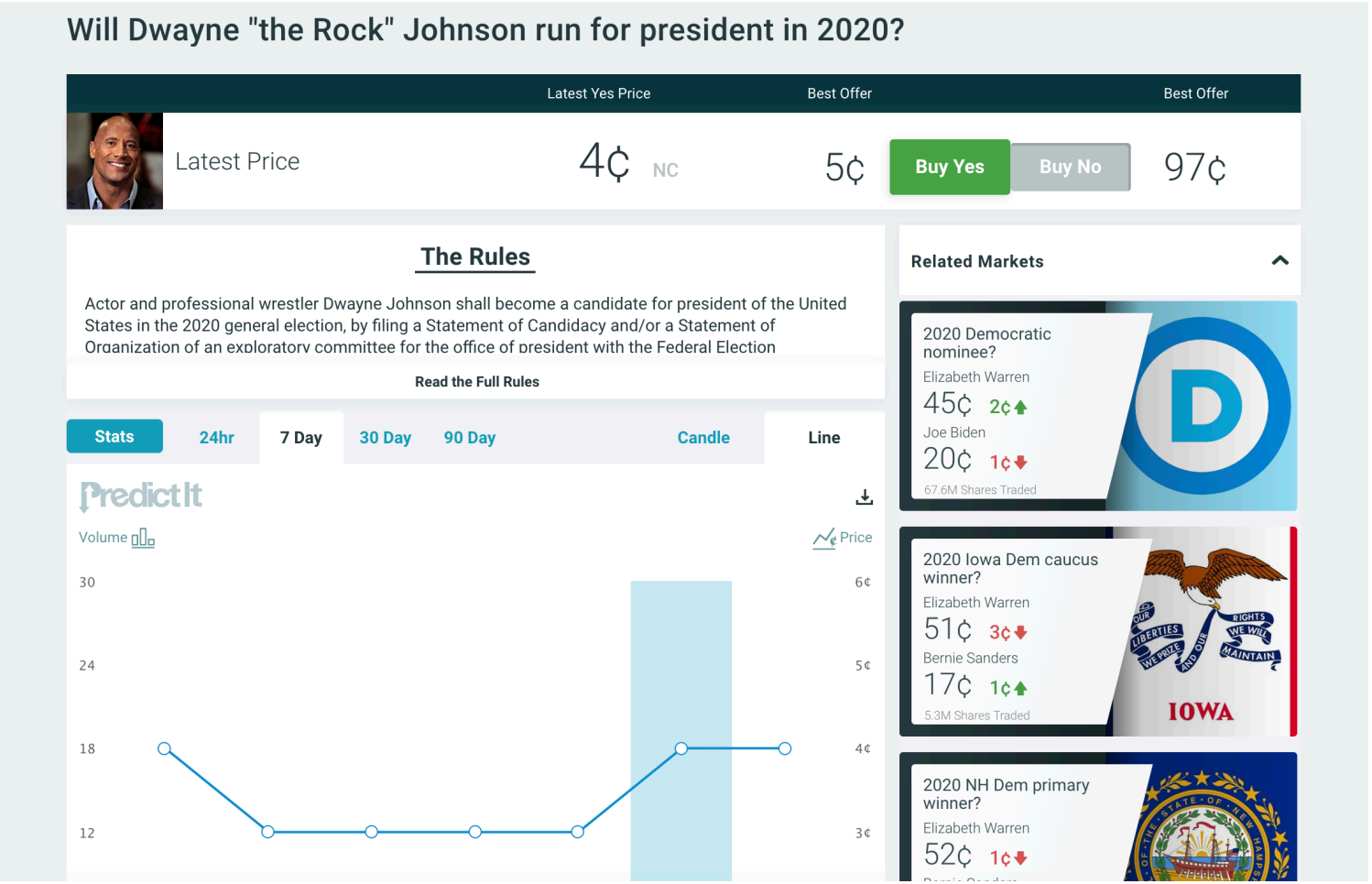
Price of a contract? $1 × percentage of shares that bet on UVA wining?

This is what we will be designing!

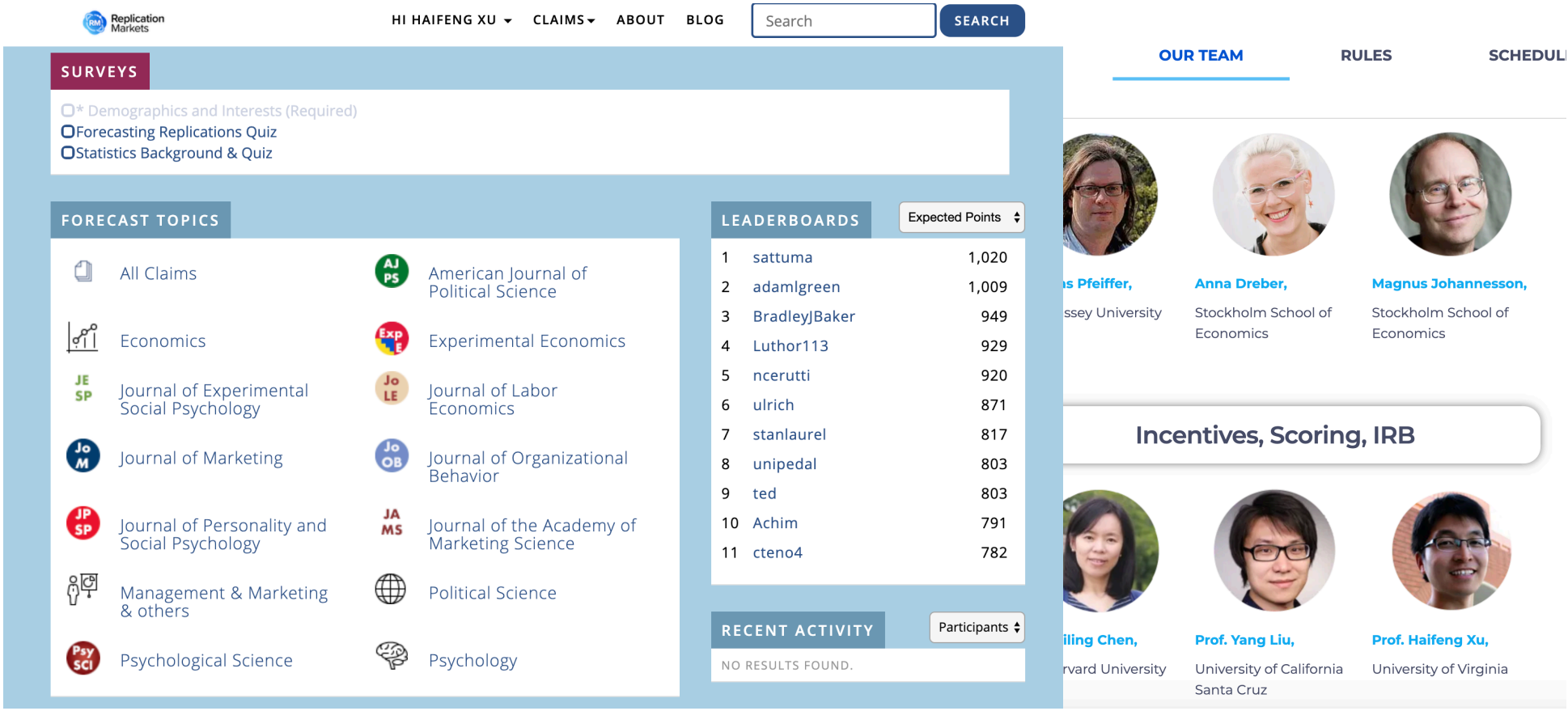
Prediction Markets: Examples



Prediction Markets: Examples

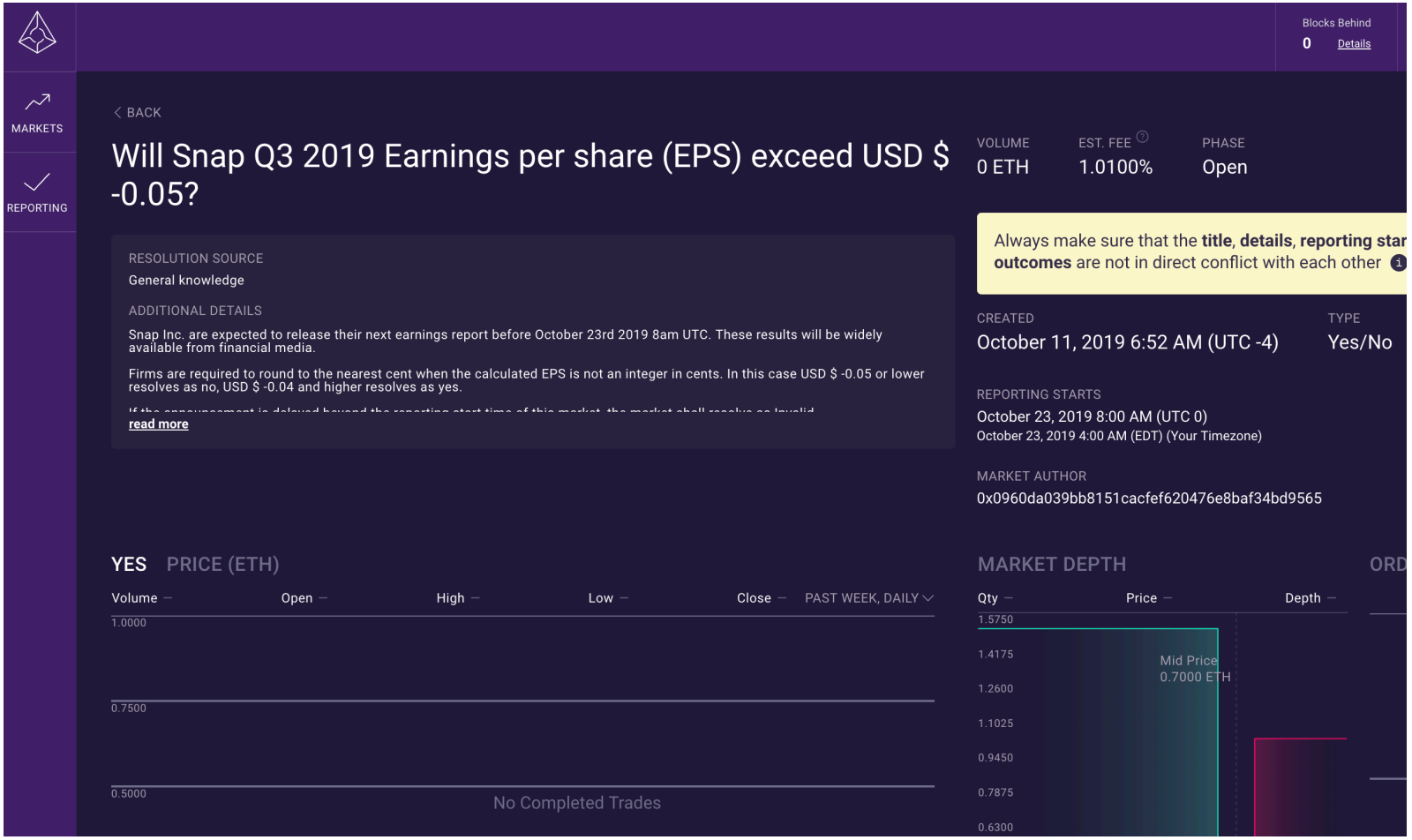


Prediction Markets: Examples



Replication Market

Prediction Markets: Examples



Augur: the first decentralized prediction markets

Does It Work?

- Yes, evidence from real markets, lab experiments, and theory
  - I.E.M. beat political polls 451/596 [Forsythe 1992, 1999][Oliven 1995][Rietz 1998][Berg 2001][Pennock 2002]
  - HP market beats sales forecast 6/8 [Plott 2000]
  - Sports betting markets provide accurate forecasts of game outcomes [Gandar 1998][Thaler 1988][Debnath EC'03][Schmidt 2002]
  - Laboratory experiments confirm information aggregation [Plott 1982;1988;1997][Forsythe 1990][Chen, EC'01]
  - Theory: "rational expectations" [Grossman 1981][Lucas 1972]
  - More …

Why Can Markets Aggregate Information?

- Price $\approx Prob(\text{event}|\text{ all information})$

$1 if UVA wins NCAA title, $0 otherwise

Why Can Markets Aggregate Information?

- Price $\approx Prob(\text{event}|\text{ all information})$

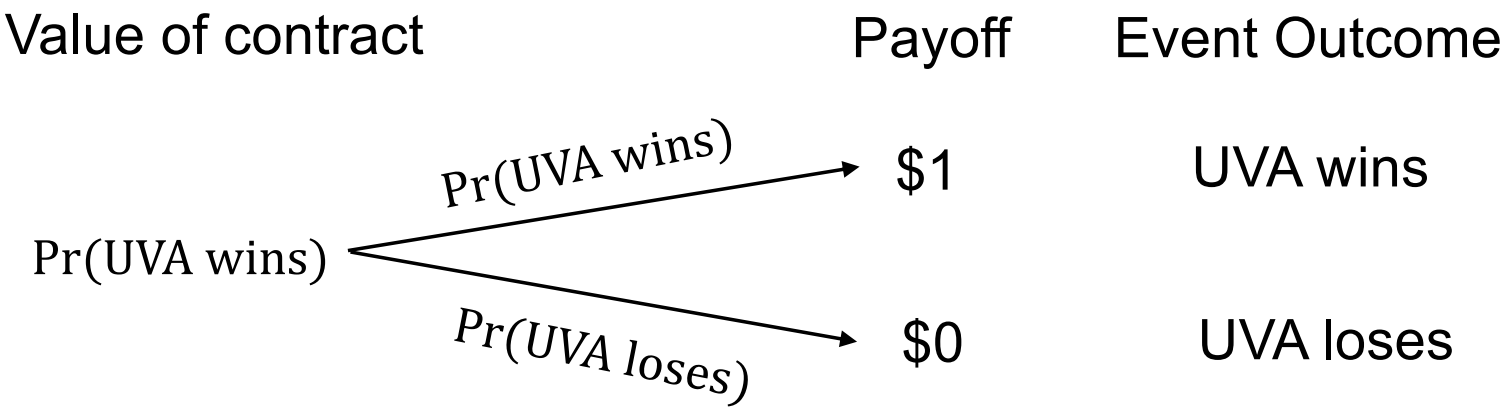
$1 if UVA wins NCAA title, $0 otherwise

Value of contract	Payoff	Event Outcome
?	$1	UVA wins
	$0	UVA loses

Why Can Markets Aggregate Information?

- Price $\approx Prob(\text{event}|\text{ all information})$

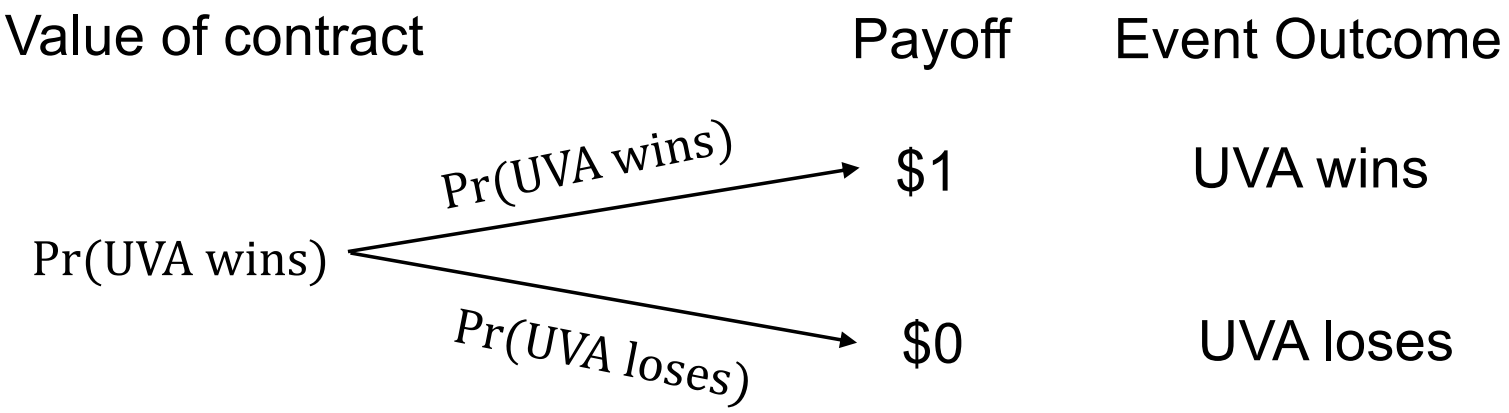
$1 if UVA wins NCAA title, $0 otherwise



Why Can Markets Aggregate Information?

- Price $\approx Prob(\text{event}|\text{ all information})$

$1 if UVA wins NCAA title, $0 otherwise



Value of contract ≈ P(UVA wins) ≈ Equilibrium price

Market Efficiency (a design goal)

Markets vs Other Prediction Approaches

Opinion Poll

- Sampling
- No incentive to be truthful
- Equally weighted information
- Hard to be real-time

Ask Experts

- Identifying experts can be hard
- Combining opinions is difficult

Prediction Markets

- Self-selection
- Monetary incentive and more
- Money-weighted information
- Real-time
- Self-organizing

Other Prediction Approaches vs Markets

Machine Learning

- Historical data
- Assume past and future are related
- Hard to incorporate recent new information

Prediction Markets

- No need for data
- No assumption on past and future
- Immediately incorporate new information

Caveat: markets have their own problems too – manipulations, irrational traders, etc.

Outline

- Introduction to Prediction Markets
- Design of Prediction Markets (PMs)
  - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates

Some Design Objectives of PMs

Liquidity: people can find counterparties to trade whenever they want

Bounded loss: total loss of the market institution is bounded

Market efficiency: Price reflects predicted probabilities.

Computational efficiency: The process of operating the market should be computationally manageable.

Continuous Double Auction (CDA) Market

$1 if UVA wins NCAA title, $0 otherwise

- Buyer orders
- Seller orders

Continuous Double Auction (CDA) Market

$1 if UVA wins NCAA title, $0 otherwise

- Buyer orders
- Seller orders

$0.12	**$0.30**

Continuous Double Auction (CDA) Market

$1 if UVA wins NCAA title, $0 otherwise

- Buyer orders
- Seller orders

Buyer orders	Seller orders
$0.12	**$0.30**
$0.09	**$0.17**

Continuous Double Auction (CDA) Market

$1 if UVA wins NCAA title, $0 otherwise

- Buyer orders
- Seller orders

Buyer orders	Seller orders
$0.12	**$0.17**
$0.09	**$0.30**

Continuous Double Auction (CDA) Market

$1 if UVA wins NCAA title, $0 otherwise

- Buyer orders

$0.12
$0.09
$0.15

- Seller orders

$0.17
$0.30
$0.13

Continuous Double Auction (CDA) Market

$1 if UVA wins NCAA title, $0 otherwise

- Buyer orders

$0.15
$0.12
$0.09

- Seller orders

$0.13
$0.17
$0.30

Continuous Double Auction (CDA) Market

$1 if UVA wins NCAA title, $0 otherwise

- Buyer orders
- Seller orders

Buyer orders		Seller orders
$0.15		**$0.13**
$0.12	Price = $0.14	**$0.17**
$0.09		**$0.30**

What's Wrong with CDA?

- Thin market problem
  - When there are not enough traders, trade may not happen.
- No trade theorem [Milgrom & Stokey 1982]
  - Why trade? These markets are zero-sum games (negative sum w/ transaction fees)
  - For all money earned, there is an equal (greater) amount lost; am I smarter than average?
  - Rational risk-neutral traders will *never* trade
  - But trade still happens …

An Alternative: Market Maker (MM)

- A market maker is the market institution who sets the prices and is willing to accept orders (buy or sell) at the price specified.
- Why? Liquidity!
- Market makers bear risk. Thus, we desire mechanisms that can bound the loss of market makers.

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- An (automated) market marker (MM)
- Sell or buy back contracts | \$1 iff e_1 | · · · | \$1 iff e_n |
- Value function ($q = (q_1, \cdots, q_n)$ is current sales quantity)

$$V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$$

Parameter b adjusts liquidity

- Price function

$$p_i(q) = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}} = \frac{\partial V(q)}{\partial q_i}$$

- To buy $x \in \mathbb{R}^n$ amount, a buyer pays: $V(q + x) - V(q)$
  - Negative x_i's mean selling contracts to MM
  - Negative payment means market maker pays the buyer
  - Market starts with $V(0) = b \log n$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event $e_1, \cdots, e_n$ is $\lambda = (\lambda_1, \cdots, \lambda_n)$, how many shares of each contract should you buy?

- Say, you buy $x \in \mathbb{R}^n$ amount
- You pay $V(q + x) - V(q)$; Your expected return is $\sum_{j \in [n]} \lambda_j \cdot x_j$
- Expected utility is

$$U(x) = \sum_{j \in [n]} \lambda_j \cdot x_j - b \log \sum_{j \in [n]} e^{(q_j + x_j)/b} + V(q)$$

- Which x maximizes your utility?

$$\frac{\partial U(x)}{\partial x_i} = \lambda_i - \frac{e^{(q_i + x_i)/b}}{\sum_{j \in [n]} e^{(q_j + x_j)/b}} = 0$$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event $e_1, \cdots, e_n$ is $\lambda = (\lambda_1, \cdots, \lambda_n)$, how many shares of each contract should you buy?

- Say, you buy $x \in \mathbb{R}^n$ amount
- You pay $V(q + x) - V(q)$; Your expected return is $\sum_{j \in [n]} \lambda_j \cdot x_j$
- Expected utility is

$$U(x) = \sum_{j \in [n]} \lambda_j \cdot x_j - b \log \sum_{j \in [n]} e^{(q_j + x_j)/b} + V(q)$$

- Which x maximizes your utility?

$$\frac{\partial U(x)}{\partial x_i} = \lambda_i - \frac{e^{(q_i + x_i)/b}}{\sum_{j \in [n]} e^{(q_j + x_j)/b}} = 0$$

The market price of contract i after your purchase

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event $e_1, \cdots, e_n$ is $\lambda = (\lambda_1, \cdots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ. Your expected utility of purchasing this amount is always non-negative.

- Why non-negative?
  - Buy 0 amount leads to 0, so optimal amount is at least as good

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q1: If your true belief of event $e_1, \cdots, e_n$ is $\lambda = (\lambda_1, \cdots, \lambda_n)$, how many shares of each contract should you buy?

Fact. The optimal amount you purchase is the amount that makes the market price equal to your belief λ. Your expected utility of purchasing this amount is always non-negative.

- This is the expected utility you believe, but may be incorrect since your λ may be inaccurate!
  - So, buy only when your prediction is really more accurate than the current market prediction
  - Achieves market efficiency: price = current best market prediction

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Q2: If market ends up with $q = (q_1, \cdots, q_n)$ shares for each contract, how much money did the MM collect?

- Answer: $V(q) - V(0) = V(q) - b \log n$
- But after event outcome is realized, MM need to pay based on contracts – what is the worst-case loss of MM?

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Value function $V(q) = b \log \sum_{j \in [n]} e^{q_j/b}$

Fact. After event outcome realizes and MM pays the contract, worst case MM loses is $b \log n$ (i.e., bounded).

Proof

- Only one event will be realized, say it is event e_i
- MM utility is

$$V(q) - b \log n - q_i$$

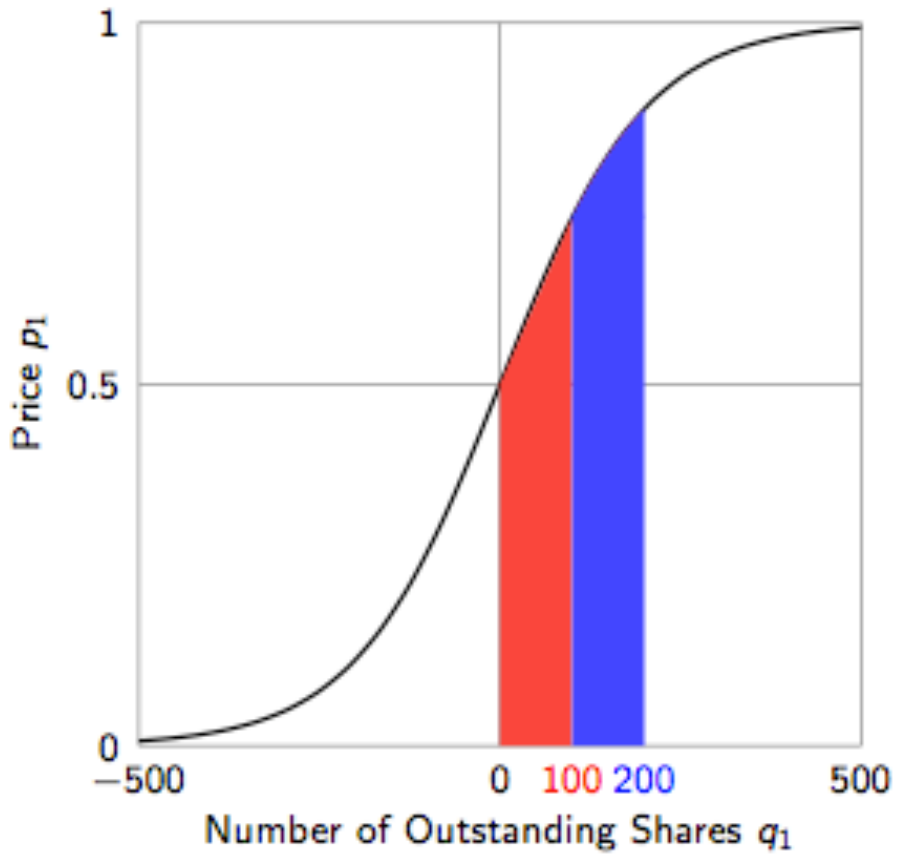
$$\geq b \log e^{q_i/b} - b \log n - q_i$$

$$\geq q_i - b \log n - q_i$$

$$\geq -b \log n$$

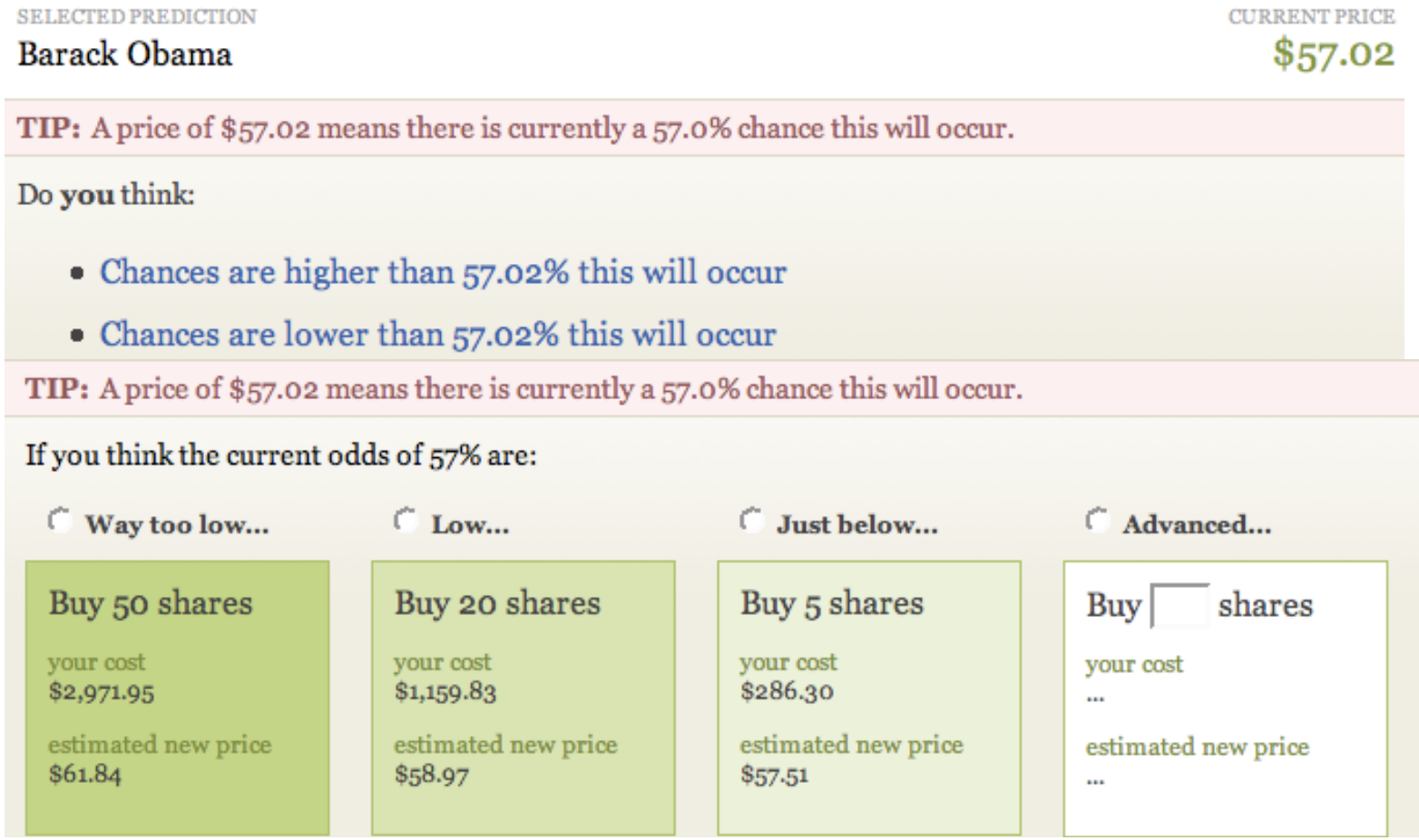
"=" can be achieved by letting $q_i \to \infty$

Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])



Example: Logarithmic Market Scoring Rule (LMSR [Hanson 03, 06])

- Has been implemented by several prediction markets
  - E.g., InklingMarkets, Washington Stock Exchange, BizPredict, Net Exchange, and (reportedly) at YooNew.



Outline

- Introduction to Prediction Markets
- Design of Prediction Markets
  - Logarithmic Market Scoring Rule (LMSR)
- LMSR and Exponential Weight Updates (EWU)

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \cdots, w_t(n)$
- Observe cost vector c_t, and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$

Action 1, $w_t(1)$

Action 2, $w_t(2)$

. . .

Action n, $w_t(n)$

$$w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}$$

$$= [w_{t-1}(i) \cdot e^{-\epsilon c_{t-1}(i)}] \cdot e^{-\epsilon c_t(i)}$$

$$= \cdots = e^{-\epsilon C_t(i)} \text{ where } C_t(i) = \sum_{\tau \le t} c_\tau(i)$$

Recap: Exponential Weight Update

- Played for T rounds; each round selects an action $i \in [n]$
- Maintains weights over n actions: $w_t(1), \cdots, w_t(n)$
- Observe cost vector c_t, and update $w_{t+1}(i) = w_t(i) \cdot e^{-\epsilon c_t(i)}, \forall i \in [n]$
- At round $t+1$, select action i with probability

$$\frac{w_t(i)}{W_t} = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

where $C_t = \sum_{\tau \leq t} c_t$ is the accumulated cost vector

This looks very much like the price function in LMSR (q is the accumulated sales quantity)

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

EWU vs LMSR

- Exponential Weight Update
  - n actions
  - Maintain weight $w_t(i)$
  - Total cost $C_T(i) = \sum_{t \le T} c_t(i)$
  - Select i with prob

$$p_i = \frac{e^{-\epsilon C_t(i)}}{\sum_{j \in [n]} e^{-\epsilon C_t(j)}}$$

  - Weights reflect how good an action is
  - Care about worst case regret

$$C_T(\text{Alg}) - \min_i C_T(i)$$

- LMSR
  - n contracts (i.e., outcomes)
  - Maintain prices $p(i)$
  - Total shares sold $q(i)$
  - Price of contract i

$$p_i = \frac{e^{q_i/b}}{\sum_{j \in [n]} e^{q_j/b}}$$

  - Prices reflect how probable is an event
  - Care about worst case MM loss

$$(\$ \text{ received}) - \min_i q(i)$$

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market markers and no-regret learning algorithms (see [Chen and Vaughan 2010])
- Markets can potentially be a very effective forecasting tool
  - Big on-going project: “replication market” for DARPA SCORE program

Remarks

- LMSR is just one particular automatic MM
- Similar relation holds for other market markers and no-regret learning algorithms (see [Chen and Vaughan 2010])
- Markets can potentially be a very effective forecasting tool
  - Big on-going project: “replication market” for DARPA SCORE program
- Mechanism design for prediction tasks
  - ML is one way but not the only way of making predictions
  - But markets and ML may augment each other

Thank You

Haifeng Xu

University of Virginia

hx4ad@virginia.edu