Announcements

- HW 3 postponed to this Thursday
- Project proposal due this Thursday as well

CS6501: Topics in Learning and Game Theory (Fall 2019)

Bayesian Persuasion

Instructor: Haifeng Xu

- Prediction markets and peer prediction study how to elicit information from others
- This lecture: when you have information, how to exploit it?
  - Relevant to mechanism design

Outline

- Introduction and Bayesian Persuasion

- Algorithms for Bayesian Persuasion

Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions

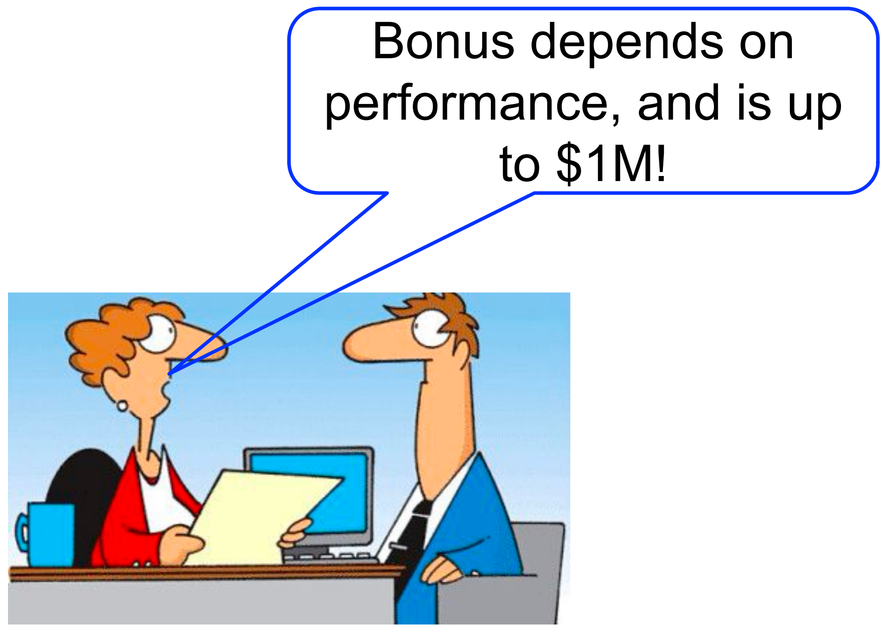
You only pay the second highest bid!

Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons

Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design



Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Mechanism Design

Two Primary Ways to Influence Agents' Behaviors

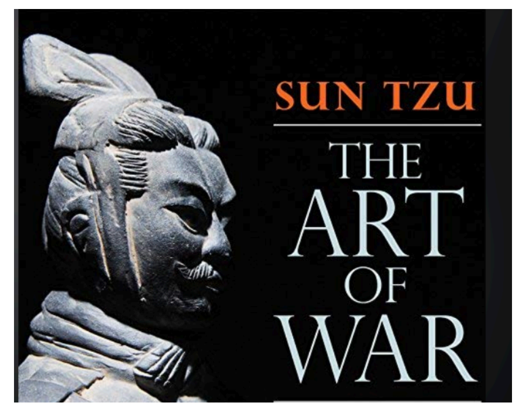
- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles

All warfare is based on deception. Hence, when we are able to attack, we must seem unable; when using our forces, we must appear inactive…

-- Sun Tzu, *The Art of War*



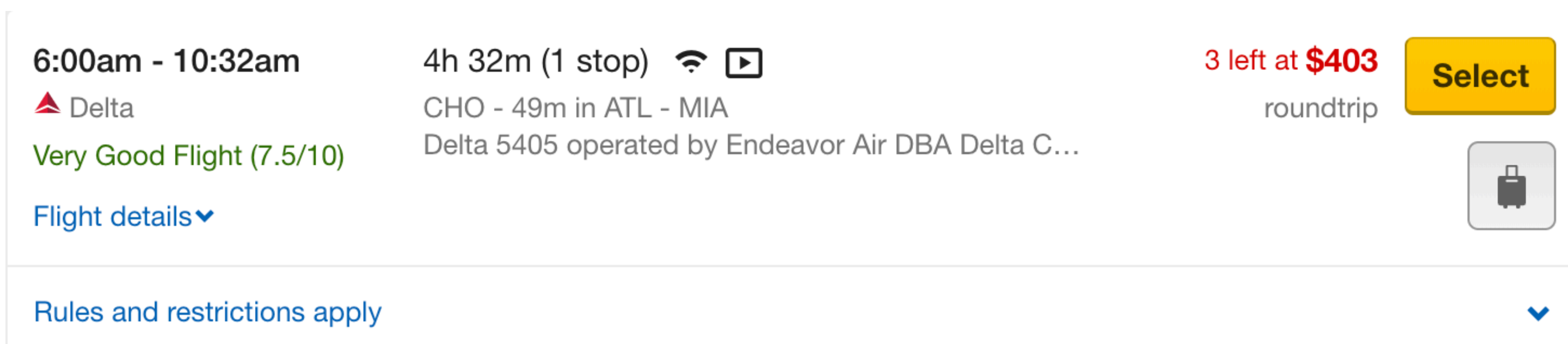
Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles
  - Strategic information disclosure

Strategic inventory information disclosure



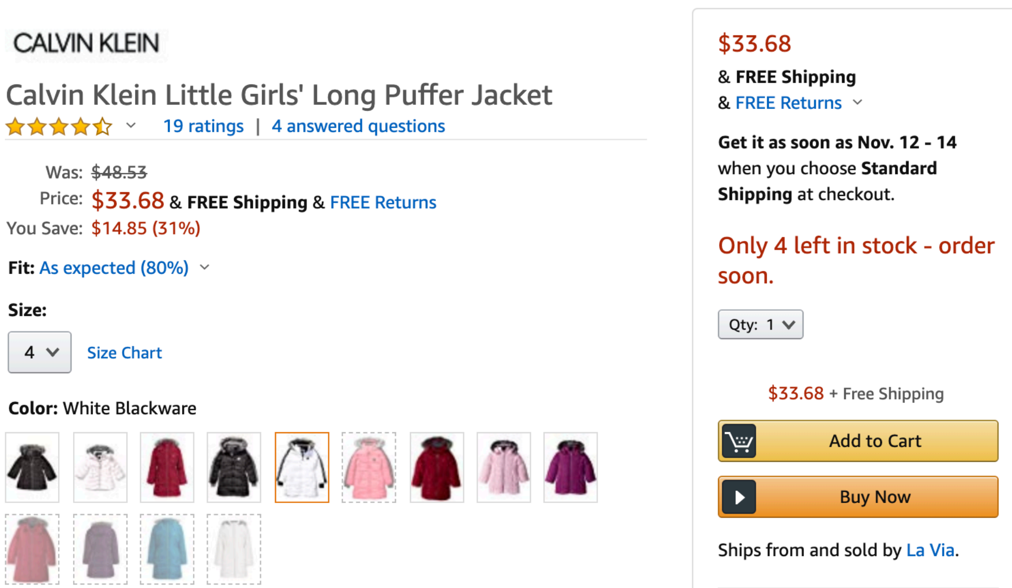
Two Primary Ways to Influence Agents' Behaviors

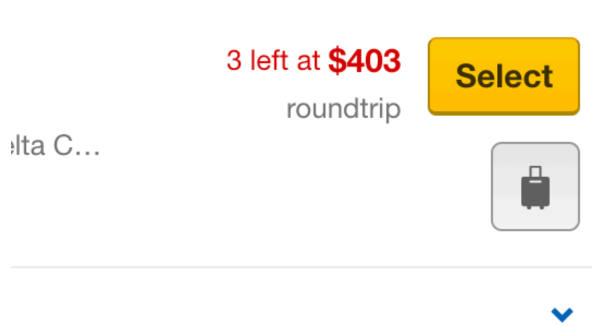
- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles
  - Strategic information disclosure

Strategic inventory information disclosure



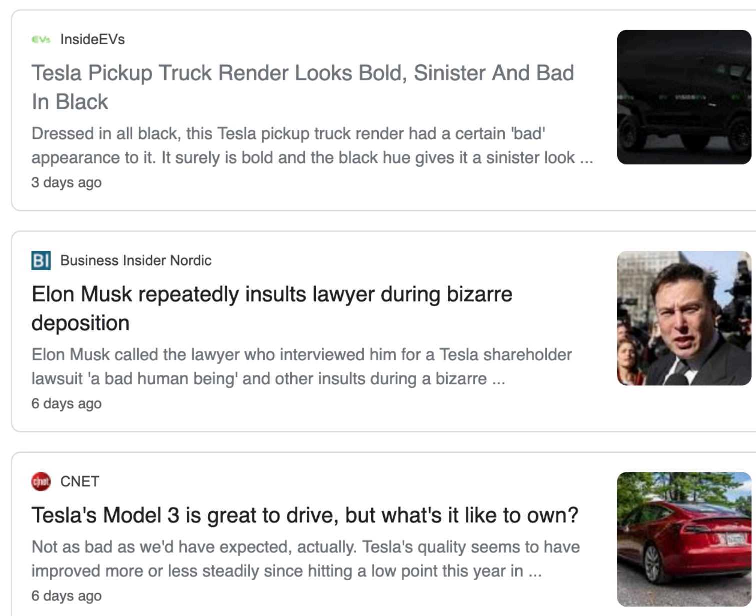


Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles
  - Strategic information disclosure
  - News articles, advertising, tweets, etc.



Two Primary Ways to Influence Agents' Behaviors

- Design/provide incentives
  - Auctions
  - Discounts/coupons
  - Job contract design

Mechanism Design

- Influence agents' beliefs
  - Deception in wars/battles
  - Strategic information disclosure
  - News articles, advertising, tweets …
  - In fact, most information you see is there for a goal

Persuasion

Persuasion is the act of exploiting an **informational advantage** in order to influence the decisions of others

- Intrinsic in human activities: advertising, negotiation, politics, security, marketing, financial regulation,…
- A large body of research

One Quarter of GDP Is Persuasion

By DONALD MCCLOSKEY AND ARJO KLAMER*

— The American Economic Review Vol. 85, No. 2, 1995.

Example: Recommendation Letters

- Advisor vs. recruiter
- 1/3 of the advisor's students are excellent; 2/3 are average
- A fresh graduate is randomly drawn from this population
- Recruiter
  - Utility $1+\epsilon$ for hiring an excellent student; -1 for an average student
  - Utility 0 for not hiring
  - A-priori, only knows the advisor's student population

$$(1+\epsilon)\times 1/3 - 1\times 2/3 \qquad < \qquad 0$$

hiring *Not hiring*

Example: Recommendation Letters

- Advisor vs. recruiter
- 1/3 of the advisor's students are excellent; 2/3 are average
- A fresh graduate is randomly drawn from this population
- Recruiter
    - Utility $1 + \epsilon$ for hiring an excellent student; -1 for an average student
    - Utility 0 for not hiring
    - A-priori, only knows the advisor's student population
- Advisor
    - Utility 1 if the student is hired, 0 otherwise
    - Knows whether the student is excellent or not

Example: Recommendation Letters

What is the advisor’s optimal “recommendation strategy”?

- Attempt 1: always say “excellent” (equivalently, no information)
  - Recruiter ignores the recommendation
  - Advisor expected utility 0

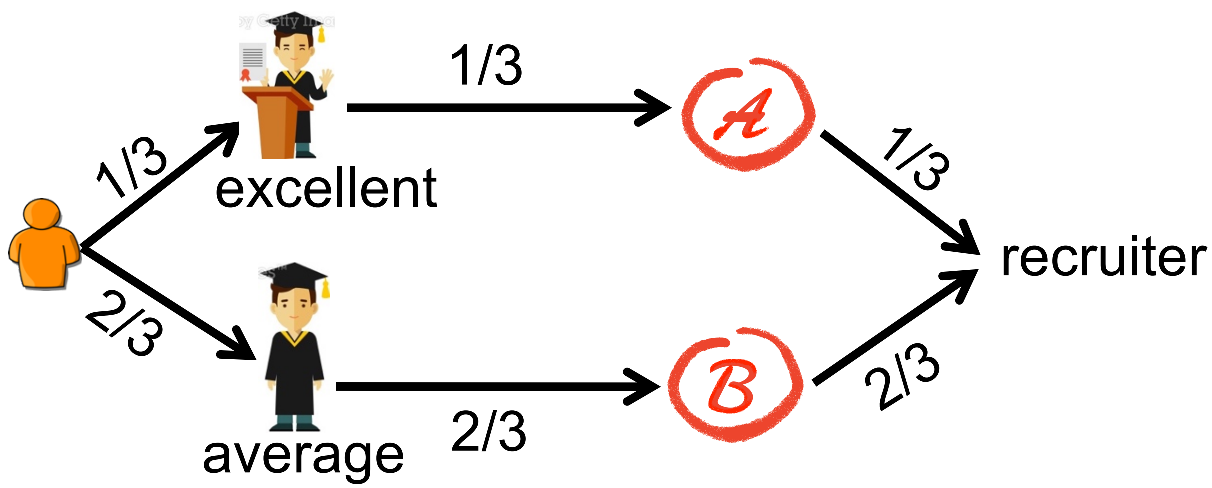
Remark

Advisor commitment: cannot deviate and recruiter knows his strategy

Example: Recommendation Letters

What is the advisor's optimal "recommendation strategy"?

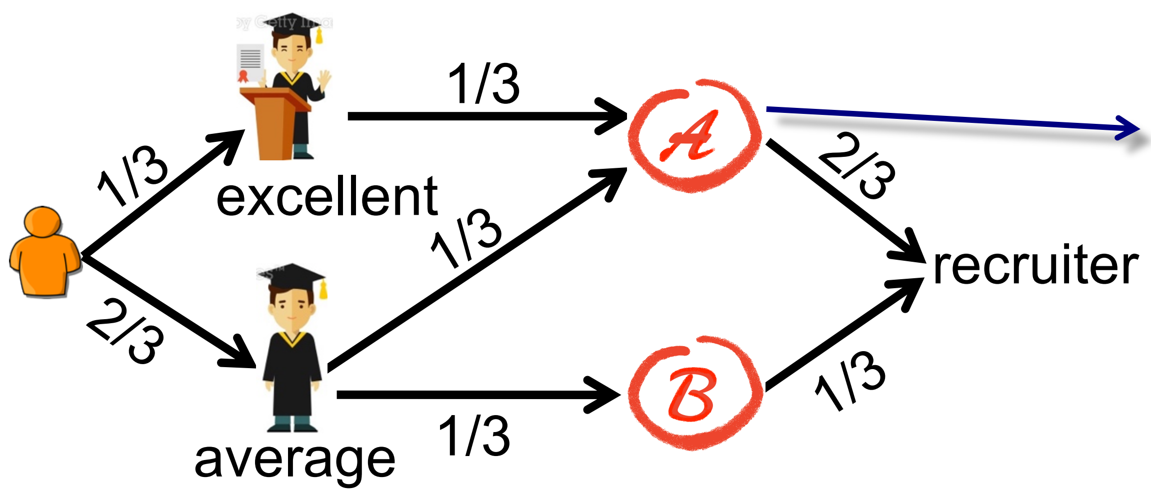
- Attempt 2: honest recommendation (i.e., full information)
  - Advisor expected utility 1/3



Example: Recommendation Letters

What is the advisor's optimal "recommendation strategy"?

- Attempt 3: noisy information → advisor expected utility 2/3



P(excellent | A) = $1/2$

$(1+\epsilon - 1)/2 \quad > \quad 0$

Hiring Not hiring

Model of Bayesian Persuasion

- Two players: persuader (Sender, she), decision maker (Receiver he)
  - Previous example: advisor = sender, recruiter = receiver
- Receiver looks to take an action $i \in [n] = \{1, 2, \ldots, n\}$
  - Receiver utility $r(i, \theta)$
  - Sender utility $s(i, \theta)$

  $\theta \in \Theta$ is a random state of nature
- Both players know $\theta \sim prior\ dist.\ \mu$, but Sender has an informational advantage – she can observe realization of θ
- Sender wants to strategically reveal info about θ to "persuade" Receiver to take an action she likes
  - Concealing or revealing all info is not necessarily the best

Well…how to reveal partial information?

Revealing Information via Signaling

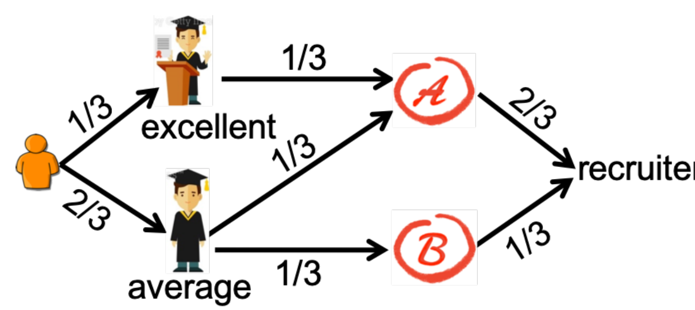
Definition: A signaling scheme is a mapping $\pi: \Theta \rightarrow \Delta_\Sigma$ where Σ is the set of all possible signals.

π is fully described by $\{\pi(\sigma,\theta)\}_{\theta\in\Theta,\sigma\in\Sigma}$ where $\pi(\sigma,\theta) =$ prob. of sending σ when observing θ (so $\sum_{\sigma\in\Sigma}\pi(\sigma,\theta) = 1$ for any θ)

Note: scheme π is always assumed public knowledge, thus known by Receiver

Example

- $\Theta = \{Excellent, Average\}, \Sigma = \{A, B\}$
- $\pi(A, Average) = 1/2$



Revealing Information via Signaling

Definition: A signaling scheme is a mapping $\pi: \Theta \to \Delta_\Sigma$ where Σ is the set of all possible signals.

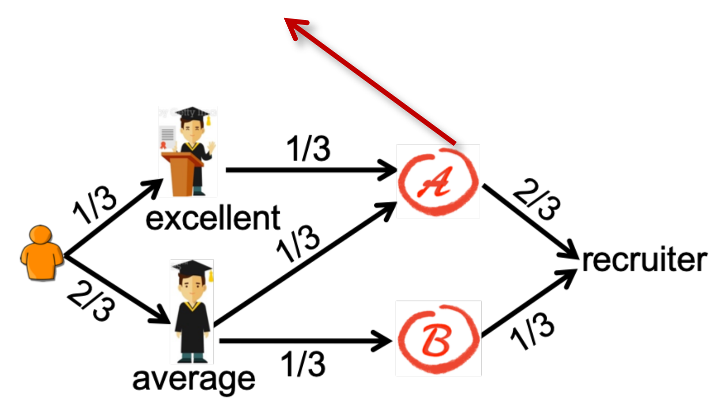
π is fully described by $\{\pi(\sigma,\theta)\}_{\theta\in\Theta,\sigma\in\Sigma}$ where $\pi(\sigma,\theta) =$ prob. of sending σ when observing θ (so $\sum_{\sigma\in\Sigma}\pi(\sigma,\theta) = 1$ for any θ)

What can Receiver infer about θ after receiving σ?

Bayes updating:

$$\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'}\pi(\sigma,\theta')\cdot\mu(\theta')}$$

$\Pr(excellent|A) = 1/2$



Revealing Information via Signaling

Definition: A signaling scheme is a mapping $\pi: \Theta \to \Delta_\Sigma$ where Σ is the set of all possible signals.

π is fully described by $\{\pi(\sigma, \theta)\}_{\theta \in \Theta, \sigma \in \Sigma}$ where $\pi(\sigma, \theta) =$ prob. of sending σ when observing θ (so $\sum_{\sigma \in \Sigma} \pi(\sigma, \theta) = 1$ for any θ)

Would such noisy information benefit Receiver?

- Expected Receiver utility conditioned on σ:

$$R(\sigma) = \max_{i \in [n]} \left[\sum_{\theta \in \Theta} r(i, \theta) \cdot \frac{\pi(\sigma,\theta) \cdot \mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta') \cdot \mu(\theta')} \right]$$

- $\Pr(\sigma) = \sum_{\theta'} \pi(\sigma, \theta') \cdot \mu(\theta')$

Revealing Information via Signaling

Definition: A signaling scheme is a mapping $\pi: \Theta \rightarrow \Delta_\Sigma$ where Σ is the set of all possible signals.

π is fully described by $\{\pi(\sigma, \theta)\}_{\theta \in \Theta, \sigma \in \Sigma}$ where $\pi(\sigma, \theta) =$ prob. of sending σ when observing θ (so $\sum_{\sigma \in \Sigma} \pi(\sigma, \theta) = 1$ for any θ)

Would such noisy information benefit Receiver?

- Expected Receiver utility conditioned on σ:

$$R(\sigma) = \max_{i \in [n]} \left[\sum_{\theta \in \Theta} r(i, \theta) \cdot \frac{\pi(\sigma,\theta) \cdot \mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta') \cdot \mu(\theta')} \right]$$

- $\Pr(\sigma) = \sum_{\theta'} \pi(\sigma, \theta') \cdot \mu(\theta')$

Revealing Information via Signaling

Definition: A signaling scheme is a mapping $\pi: \Theta \rightarrow \Delta_\Sigma$ where Σ is the set of all possible signals.

π is fully described by $\{\pi(\sigma, \theta)\}_{\theta \in \Theta, \sigma \in \Sigma}$ where $\pi(\sigma, \theta) =$ prob. of sending σ when observing θ (so $\sum_{\sigma \in \Sigma} \pi(\sigma, \theta) = 1$ for any θ)

Would such noisy information benefit Receiver?

- Expected Receiver utility conditioned on σ:

$$R(\sigma) = \max_{i \in [n]} \left[\sum_{\theta \in \Theta} r(i, \theta) \cdot \frac{\pi(\sigma,\theta) \cdot \mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta') \cdot \mu(\theta')} \right]$$

- $\Pr(\sigma) = \sum_{\theta'} \pi(\sigma, \theta') \cdot \mu(\theta')$
  - $\Pr(\sigma) \cdot R(\sigma) = \max_i \sum_{\theta \in \Theta} r(i, \theta) \cdot \pi(\sigma, \theta) \cdot \mu(\theta)$

- Expected Receiver utility under π: $\sum_\sigma \max_i \sum_{\theta \in \Theta} r(i, \theta) \cdot \pi(\sigma, \theta) \cdot \mu(\theta)$

Revealing Information via Signaling

Fact. Receiver's expected utility (weakly) increases under any signaling scheme π.

Revealing Information via Signaling

Fact. Receiver's expected utility (weakly) increases under any signaling scheme π.

Proof:

- Expected Receiver utility under π: $\sum_\sigma \max_i \sum_{\theta \in \Theta} r(i,\theta) \cdot \pi(\sigma,\theta) \cdot \mu(\theta)$
- Expected Receiver utility without information: $\max_i \sum_{\theta \in \Theta} r(i,\theta) \cdot \mu(\theta)$

Revealing Information via Signaling

Fact. Receiver's expected utility (weakly) increases under any signaling scheme π.

Proof:

- Expected Receiver utility under π: $\sum_\sigma \max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma,\theta)\cdot\mu(\theta)$
- Expected Receiver utility without information: $\max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\mu(\theta)$
- Let $i^* = \arg\max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\mu(\theta)$, we have

$$\begin{aligned}\sum_\sigma \max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma,\theta)\cdot\mu(\theta) &\geq \sum_\sigma \sum_{\theta\in\Theta} r(i^*,\theta)\cdot\pi(\sigma,\theta)\cdot\mu(\theta)\\ &= \sum_{\theta\in\Theta} r(i^*,\theta)\cdot\left[\sum_\sigma \pi(\sigma,\theta)\right]\cdot\mu(\theta)\\ &= \sum_{\theta\in\Theta} r(i^*,\theta)\cdot\mu(\theta)\end{aligned}$$

Revealing Information via Signaling

Fact. Receiver's expected utility (weakly) increases under any signaling scheme π.

Proof:

- Expected Receiver utility under π: $\sum_\sigma \max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma,\theta)\cdot\mu(\theta)$
- Expected Receiver utility without information: $\max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\mu(\theta)$
- Let $i^* = \arg\max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\mu(\theta)$, we have

$$\begin{aligned}\sum_\sigma \max_i \sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma,\theta)\cdot\mu(\theta) &\geq \sum_\sigma \sum_{\theta\in\Theta} r(i^*,\theta)\cdot\pi(\sigma,\theta)\cdot\mu(\theta)\\ &= \sum_{\theta\in\Theta} r(i^*,\theta)\cdot\left[\sum_\sigma \pi(\sigma,\theta)\right]\cdot\mu(\theta)\\ &= \sum_{\theta\in\Theta} r(i^*,\theta)\cdot\mu(\theta)\end{aligned}$$

Revealing Information via Signaling

Fact. Receiver's expected utility (weakly) increases under any signaling scheme π.

Remarks:

- Signaling scheme does increase Receiver's utility
- More (even noisy) information always helps a decision maker (DM)
  - Note true if multiple DMs (will see examples later)

Revealing Information via Signaling

Fact. Receiver's expected utility (weakly) increases under any signaling scheme π.

Remarks:

- ➢ Signaling scheme does increase Receiver's utility
- ➢ More (even noisy) information always helps a decision maker (DM)
  - • Note true if multiple DMs (will see examples later)

Corollary. Receiver's expected utility is maximized when Sender reveals full info, i.e., directly revealing the realized θ.

Because any other noisy scheme π can be improved by further revealing θ itself

Revealing Information via Signaling

Fact. Receiver's expected utility (weakly) increases under any signaling scheme π.

Remarks:

- Signaling scheme does increase Receiver's utility
- More (even noisy) information always helps a decision maker (DM)
  - Note true if multiple DMs (will see examples later)

Corollary. Receiver's expected utility is maximized when Sender reveals full info, i.e., directly revealing the realized θ.

But this is not Sender's goal…

Sender Objective: carefully pick π to maximize her expected utility

Outline

- Introduction and Bayesian Persuasion
- Algorithms for Bayesian Persuasion

Q: What worries you the most when designing $\pi = \{\pi(\theta, \sigma)\}_{\theta \in \Theta, \sigma \in \Sigma}$?

- Don't know what is the set of all possible signals Σ...
- Like in mechanism design, too many signals to consider in this world
  - Again, you can use "looking 45° up to the sky" as a signal
- Key observation: a signal is mathematically nothing but a posterior distribution over Θ
  - Recall the Bayes updates: $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta) \cdot \mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta') \cdot \mu(\theta')}$
- It turns out that n signals suffice

Revelation Principle

Fact. There always exists an optimal signaling scheme that uses at most $n(=\#$ receiver actions) signals, where signal σ_i induce optimal Receiver action i

Revelation Principle

Fact. There always exists an optimal signaling scheme that uses at most $n (= \#$ receiver actions) signals, where signal σ_i induce optimal Receiver action i

- Conditioned on any signal σ
  - Receiver infers $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta')\cdot\mu(\theta')}$
  - Receiver takes optimal action $i^* = \arg\max_{i\in[n]} \sum_\theta \Pr(\theta|\sigma)\, r(i,\theta)$

Revelation Principle

Fact. There always exists an optimal signaling scheme that uses at most $n(=\#$ receiver actions) signals, where signal σ_i induce optimal Receiver action i

- Conditioned on any signal σ
  - Receiver infers $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta')\cdot\mu(\theta')}$
  - Receiver takes optimal action $i^* = \arg\max_{i\in[n]} \sum_{\theta} \Pr(\theta|\sigma)\, r(i,\theta)$
- Now, if signal σ and σ' result in the same optimal action i^*, Sender can instead send a new signal $\sigma_{i^*} = (\sigma, \sigma')$ in both cases

Revelation Principle

Fact. There always exists an optimal signaling scheme that uses at most $n(=\#$ receiver actions) signals, where signal σ_i induce optimal Receiver action i

- Conditioned on any signal σ
  - Receiver infers $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'}\pi(\sigma,\theta')\cdot\mu(\theta')}$
  - Receiver takes optimal action $i^* = \arg\max_{i\in[n]} \sum_\theta \Pr(\theta|\sigma)\, r(i,\theta)$
- Now, if signal σ and σ' result in the same optimal action i^*, Sender can instead send a new signal $\sigma_{i^*} = (\sigma,\sigma')$ in both cases
  - Claim: i^* is still the optimal action conditioned on σ_{i^*}

$$\sum_\theta \Pr(\theta|\sigma)\, r(i^*,\theta) \geq \sum_\theta \Pr(\theta|\sigma)\, r(i,\theta), \quad \forall i$$

$$\sum_\theta \Pr(\theta|\sigma')\, r(i^*,\theta) \geq \sum_\theta \Pr(\theta|\sigma')\, r(i,\theta), \quad \forall i$$

$$\Rightarrow \quad \sum_\theta[\Pr(\theta|\sigma)p + \Pr(\theta|\sigma')(1-p)]r(i^*,\theta) \geq \sum_\theta[\Pr(\theta|\sigma)p + \Pr(\theta|\sigma')(1-p)]r(i,\theta), \quad \forall i$$

$\Pr(\theta|\sigma^*)$ is a convex combination of $\Pr(\theta|\sigma)$ and $\Pr(\theta|\sigma')$

Revelation Principle

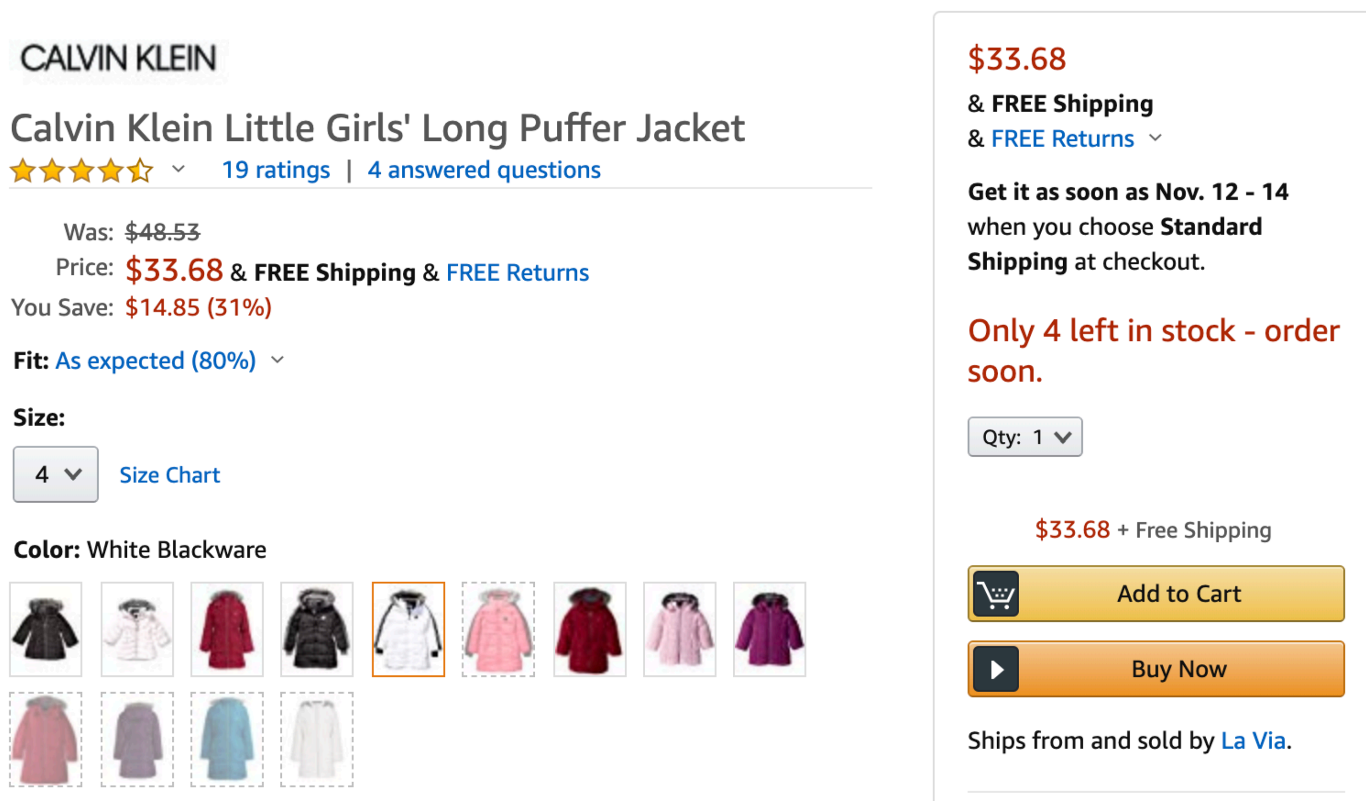
Fact. There always exists an optimal signaling scheme that uses at most n(= # receiver actions) signals, where signal σ_i induce optimal Receiver action i

- Conditioned on any signal σ
  - Receiver infers $\Pr(\theta|\sigma) = \frac{\pi(\sigma,\theta)\cdot\mu(\theta)}{\sum_{\theta'} \pi(\sigma,\theta')\cdot\mu(\theta')}$
  - Receiver takes optimal action $i^* = \arg\max_{i\in[n]} \sum_\theta \Pr(\theta|\sigma)\, r(i,\theta)$
- Now, if signal σ and σ' result in the same optimal action i^*, Sender can instead send a new signal $\sigma_{i^*} = (\sigma,\sigma')$ in both cases
  - Claim: i^* is still the optimal action conditioned on σ_{i^*}
  - Both players' utilities did not change as receiver still takes i^* as Sender wanted
- Can merge all signals with optimal receiver action i^* as a single signal σ_{i^*}

Revelation Principle

Fact. There always exists an optimal signaling scheme that uses at most n(= # receiver actions) signals, where signal σ_i induce optimal Receiver action i

➢ Each σ_i can be viewed as an action recommendation of i



Optimal Persuasion via Linear Program

- Input: prior μ, sender payoff $s(i,\theta)$, receiver payoff $r(i,\theta)$
- Variables: $\pi(\sigma_i,\theta)$

Sender expected utility
(we know Receiver will take i at signal σ_i)

$$
\begin{array}{lll}
\max & \boxed{\sum_{\theta\in\Theta}\sum_{i=1}^{n} s(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta)} & \\
\text{s.t.} & \sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) \geq \sum_{\theta\in\Theta} r(j,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta), & \text{for } i,j\in[n]. \\
& \sum_{i=1}^{n}\pi(\sigma_i,\theta) = 1, & \text{for } \theta\in\Theta. \\
& \pi(\sigma_i,\theta)\geq 0, & \text{for } \theta\in\Theta, i\in[n].
\end{array}
$$

Optimal Persuasion via Linear Program

- Input: prior μ, sender payoff $s(i,\theta)$, receiver payoff $r(i,\theta)$
- Variables: $\pi(\sigma_i,\theta)$

σ_i indeed incentivizes Receiver best action i

$$
\begin{array}{lll}
\max & \sum_{\theta\in\Theta}\sum_{i=1}^{n} s(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) & \\
\text{s.t.} & \boxed{\sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) \geq \sum_{\theta\in\Theta} r(j,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta),} & \text{for } i,j\in[n]. \\
 & \sum_{i=1}^{n}\pi(\sigma_i,\theta) = 1, & \text{for } \theta\in\Theta. \\
 & \pi(\sigma_i,\theta)\geq 0, & \text{for } \theta\in\Theta, i\in[n].
\end{array}
$$

Optimal Persuasion via Linear Program

- Input: prior μ, sender payoff $s(i, \theta)$, receiver payoff $r(i, \theta)$
- Variables: $\pi(\sigma_i, \theta)$

$$
\begin{array}{lll}
\max & \sum_{\theta\in\Theta}\sum_{i=1}^{n} s(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) & \\
\text{s.t.} & \sum_{\theta\in\Theta} r(i,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta) \geq \sum_{\theta\in\Theta} r(j,\theta)\cdot\pi(\sigma_i,\theta)\mu(\theta), & \text{for } i,j\in[n]. \\
 & \sum_{i=1}^{n}\pi(\sigma_i,\theta)=1, & \text{for } \theta\in\Theta. \\
 & \pi(\sigma_i,\theta)\geq 0, & \text{for } \theta\in\Theta, i\in[n].
\end{array}
$$

π is a valid signaling scheme

Thank You

Haifeng Xu

University of Virginia

hx4ad@virginia.edu