Announcements

- HW 3 and proposal due today

CS6501: Topics in Learning and Game Theory
(Fall 2019)

Selling Information

Instructor: Haifeng Xu

Outline

- Bayesian Persuasion and Information Selling

- Sell to a Single Decision Maker

- Sell to Multiple Decision Makers

Recap: Bayesian Persuasion

Persuasion is the act of exploiting an informational advantage in order to influence the decisions of others

- One of the two primarily ways to influence agents’ behaviors
  - Another way is through designing incentives
- Accounts for a significant share in economic activities
  - Advertising, marketing, security, investment, financial regulation,…

The Bayesian Persuasion Model

- Two players: a sender (she) and a receiver (he)
  - Sender has information, receiver is a decision maker
- Receiver takes an action $i \in [n] = \{1,2,\cdots,n\}$
  - Receiver utility $r(i,\theta)$ and sender utility $s(i,\theta)$
  - $\theta \sim prior\ dist. p$ is a random state of nature
- Both players know prior p, but sender additionally observes θ

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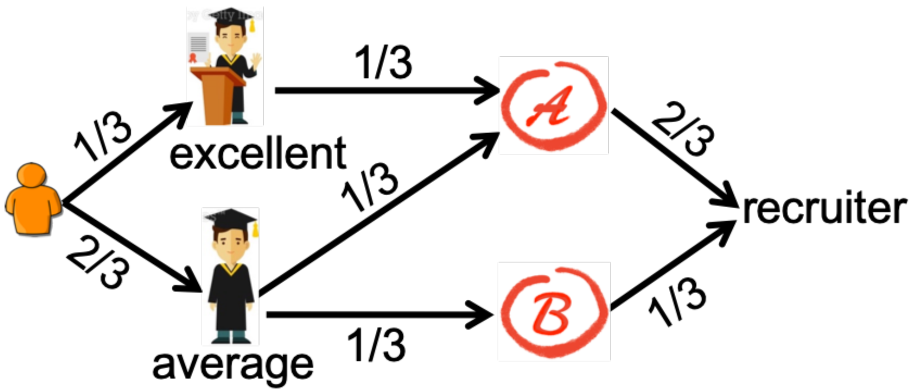
- Both players know prior p, but sender additionally observes θ
- Sender reveals partial information via a signaling scheme to influence receiver's decision and maximize her utility

Definition: A signaling scheme is a mapping $\pi: \Theta \to \Delta_\Sigma$ where Σ is the set of all possible signals.

π is fully described by $\{\pi(\sigma,\theta)\}_{\theta\in\Theta,\sigma\in\Sigma}$ where $\pi(\sigma,\theta) =$ prob. of sending σ when observing θ (so $\sum_{\sigma\in\Sigma}\pi(\sigma,\theta) = 1$ for any θ)

Example: Recommendation Letters

- Sender = advisor, receiver = recruiter
- $\Theta = \{excellent, average\}, \mu(excellent) = 1/3$
- Receiver decides Hire or NotHire
  - Results in utilities for receiver and sender
- Optimal strategy is a signaling scheme



Optimal Signaling via Linear Program

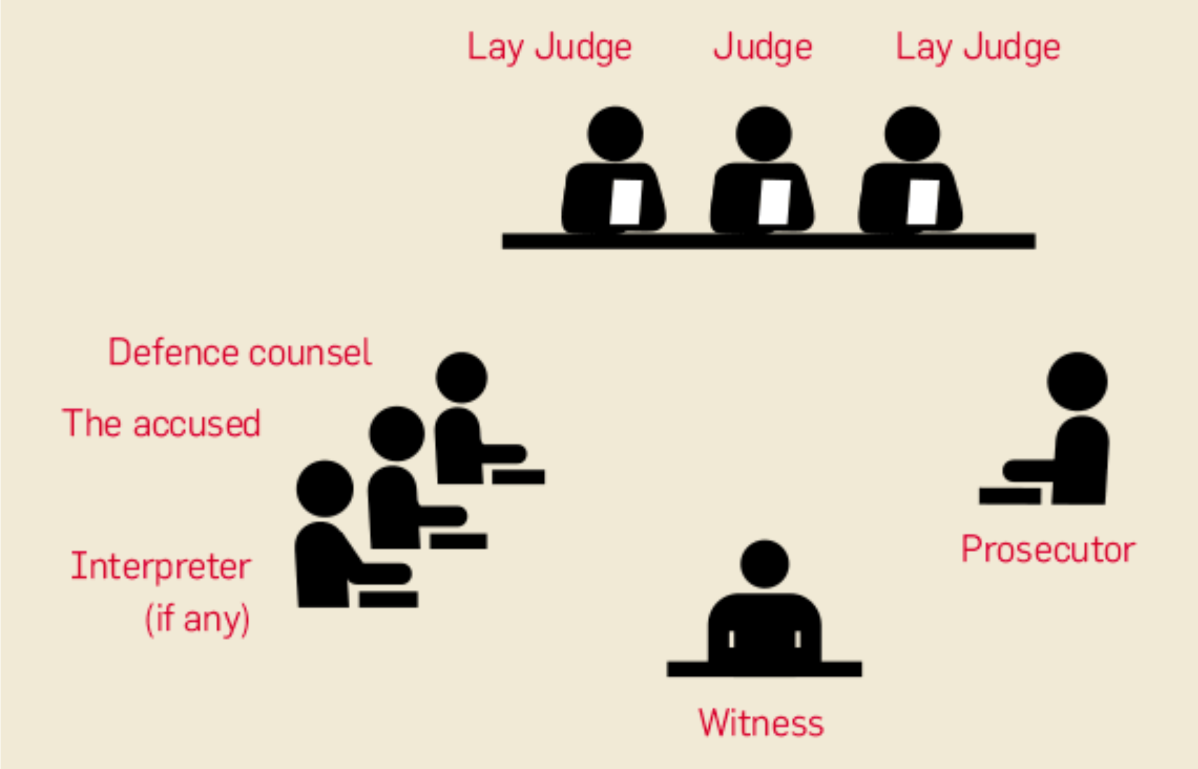
Revelation Principle. There always exists an optimal signaling scheme that uses at most $n(=\#$ receiver actions) signals, where signal σ_i induce optimal receiver action i

- Optimal signaling scheme is computed by an LP
  - Variables: $\pi(\sigma_i, \theta) =$ prob of sending σ_i conditioned on θ
  - Send $\sigma_i =$ recommend action i

$$\begin{array}{lll} \max & \sum_{\theta \in \Theta} \sum_{i=1}^{n} s(i, \theta) \cdot \pi(\sigma_i, \theta) p(\theta) & \\ \text{s.t.} & \sum_{\theta \in \Theta} r(i, \theta) \cdot \pi(\sigma_i, \theta) p(\theta) \geq \sum_{\theta \in \Theta} r(j, \theta) \cdot \pi(\sigma_i, \theta) p(\theta), & \text{for } i, j \in [n]. \\ & \sum_{i=1}^{n} \pi(\sigma_i, \theta) = 1, & \text{for } \theta \in \Theta. \\ & \pi(\sigma_i, \theta) \geq 0, & \text{for } \theta \in \Theta, i \in [n]. \end{array}$$

Many Other Examples and Extensions

- Prosecutor persuades judge



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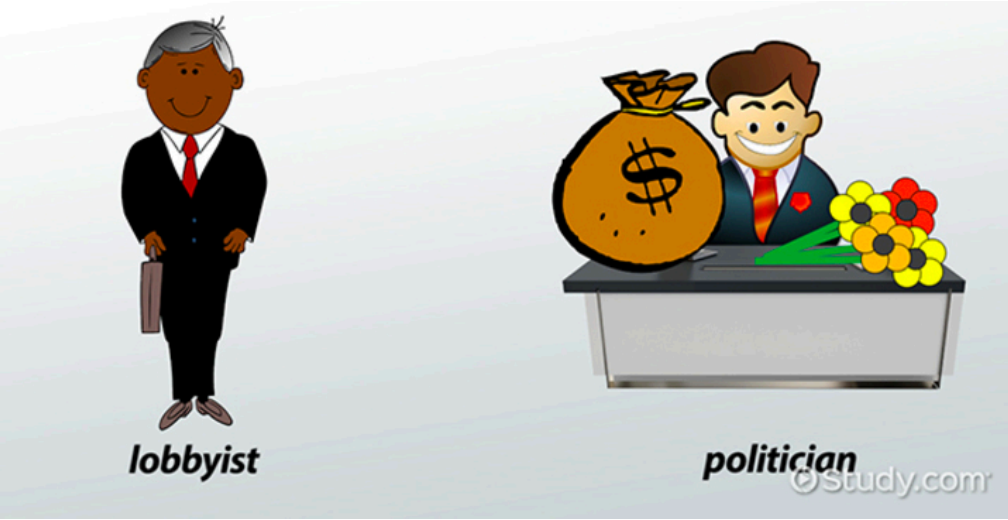

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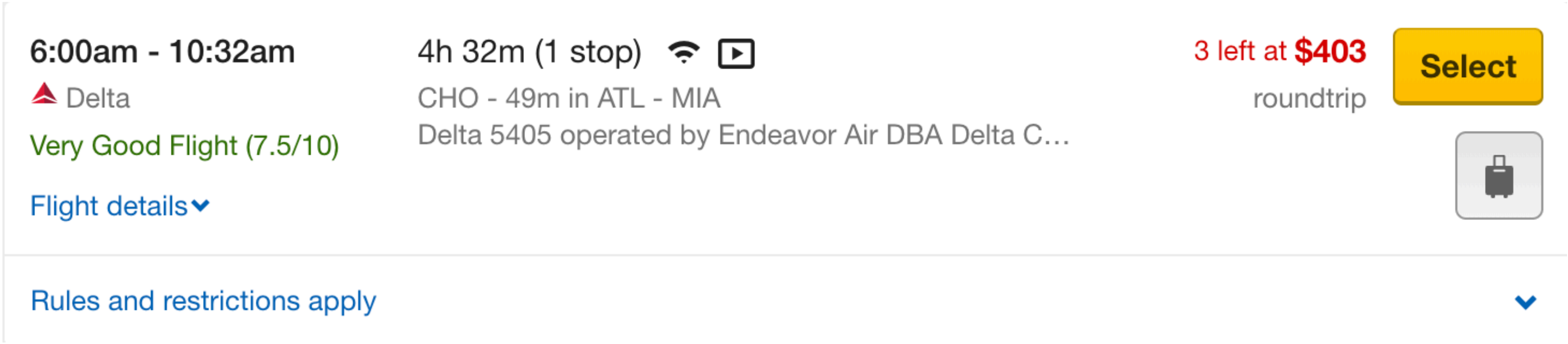

6:00am - 10:32am

Delta

Very Good Flight (7.5/10)

4h 32m (1 stop)

CHO - 49m in ATL - MIA

Delta 5405 operated by Endeavor Air DBA Delta C…

3 left at **$403**

roundtrip

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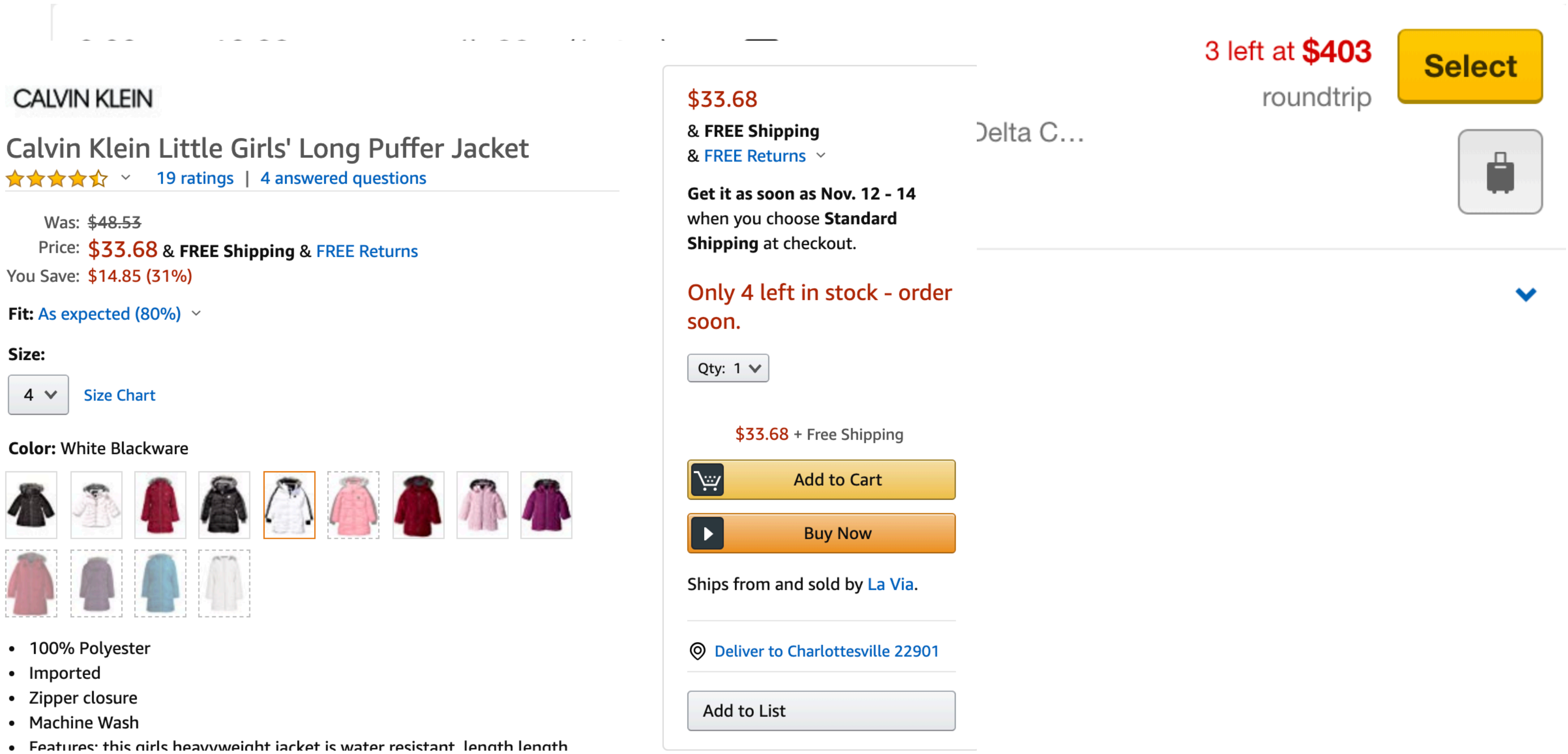

3 left at **$403**

roundtrip

CALVIN KLEIN

Calvin Klein Little Girls' Long Puffer Jacket

19 ratings | 4 answered questions

Was: ~~$48.53~~

Price: $33.68 & **FREE Shipping** & FREE Returns

You Save: $14.85 (31%)

Fit: As expected (80%)

Size: 4

Color: White Blackware

- 100% Polyester
- Imported
- Zipper closure
- Machine Wash
- Features: this girls heavyweight jacket is water resistant, length length

$33.68

& **FREE Shipping**

& FREE Returns

Get it as soon as Nov. 12 - 14 when you choose **Standard Shipping** at checkout.

Only 4 left in stock - order soon.

Qty: 1

$33.68 + Free Shipping

Ships from and sold by La Via.

Deliver to Charlottesville 22901

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- ➢Prosecutor persuades judge
- ➢Lobbyists persuade politicians
- ➢Election candidates persuade voters
- ➢Sellers persuade buyers
- ➢Executives persuade stockholders

Elon Musk @elonmusk · Oct 16

Tesla wins best headlights award 🥇 twitter.com/Teslarati/stat…

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- . . .

Many persuasion models built upon Bayesian persuasion

- Persuading many receivers, voters, attackers, drivers on road network, buyers in auctions, etc..
- Private vs public persuasion
- Selling information is also a variant

Selling Information – the Basic Model

- Sender = seller, Receiver = buyer who is a decision maker
- Buyer takes an action $i \in [n] = \{1, \cdots, n\}$
- Buyer has a utility function $u(i, \theta; \omega)$ where
  - $\theta \sim dist.\ p$ is a random state of nature
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Remarks:

- u, p, f are public knowledge
- Assume θ, ω are independent
- In mechanism design, seller also does not know buyer's value

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Q: How to price the item if seller knowns buyer's value of it?

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  - $\omega \sim dist. f$ captures buyer's (private) utility type
- Seller observes the state θ; Buyer knows his private type ω
- Seller would like to sell her information about θ to maximize revenue

Key differences from Bayesian persuasion

- Seller does not have a utility fnc – instead maximize revenue
- Buyer here has private info ω, which is unknown to seller

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Warm-up: What if Buyer Has no Private Info

- $u(i, \theta; \omega)$ where sate $\theta \sim dist.\, p$ and buyer type $\omega \sim dist. f$
- When seller also observes ω . . .

Q: How to sell information optimally?

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- How to charge the most?
  - Reveal full information helps the buyer the most. Why?
  - So OPT is to charge him following amount and then reveal θ directly

$$\text{Payment} = \sum_{\theta \in \Theta} p(\theta) \cdot [\max_i u(i, \theta; \omega)] - \max_i \sum_{\theta \in \Theta} p(\theta) \cdot u(i, \theta; \omega)$$

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Buyer expected utility if learns θ

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Buyer expected utility without knowing θ

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More interesting and realistic is when buyer has private info

Sell Information: Challenge 1

The class of mechanisms is too broad

- The mechanism will: (1) elicit private info from buyer; (2) reveal info based on realized θ; (3) charge buyer
- May interact with buyer for many rounds
- Buyer may misreport his private info of ω

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Theorem (Revelation Principle). Any information selling mechanism can be "simulated" by a direct and truthful revelation mechanism:

1. Ask buyer to report ω
2. Charge buyer t_ω and reveal info to buyer via signaling scheme π_ω

- Proof: similar to proof of revelation principle for mechanism design
- Optimal mechanism reduces to an incentive compatible menu $\{t_\omega, \pi_\omega\}_\omega$

Sell Information: Challenge 2

Signaling scheme π_ω is still complicated

- For any fixed buyer type ω, how many signals needed for π_ω?
  - Still n signals with σ_i recommending action i?
  - Previous argument of merging all signals with same buyer ω best response is not valid any more – why?

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So merging signals in π_ω retains this constraint

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Key idea: this term will only decrease since ω' gets less info due to merging of signals

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Theorem (Simplifying Signaling Schemes). There always exists an optimal incentive compatible menu $\{t_\omega, \pi_\omega\}_\omega$, such that π_ω uses at most n signals with σ_i recommending action i

Such an information-selling mechanism is like consulting – buyer reports type ω, seller charges him t_ω

Sell Information: the Optimal Mechanism

The Consulting Mechanism

1. Elicit buyer type ω
2. Charge buyer t_ω
3. Observe realized state θ and recommend action i to the buyer with probability $\pi_\omega(\sigma_i, \theta)$

- Will be incentive compatible – reporting true ω is optimal
- The recommended action is guaranteed to be the optimal action for buyer ω given his information
- $\{t_\omega, \pi_\omega\}_\omega$ is public knowledge, and computed by LP

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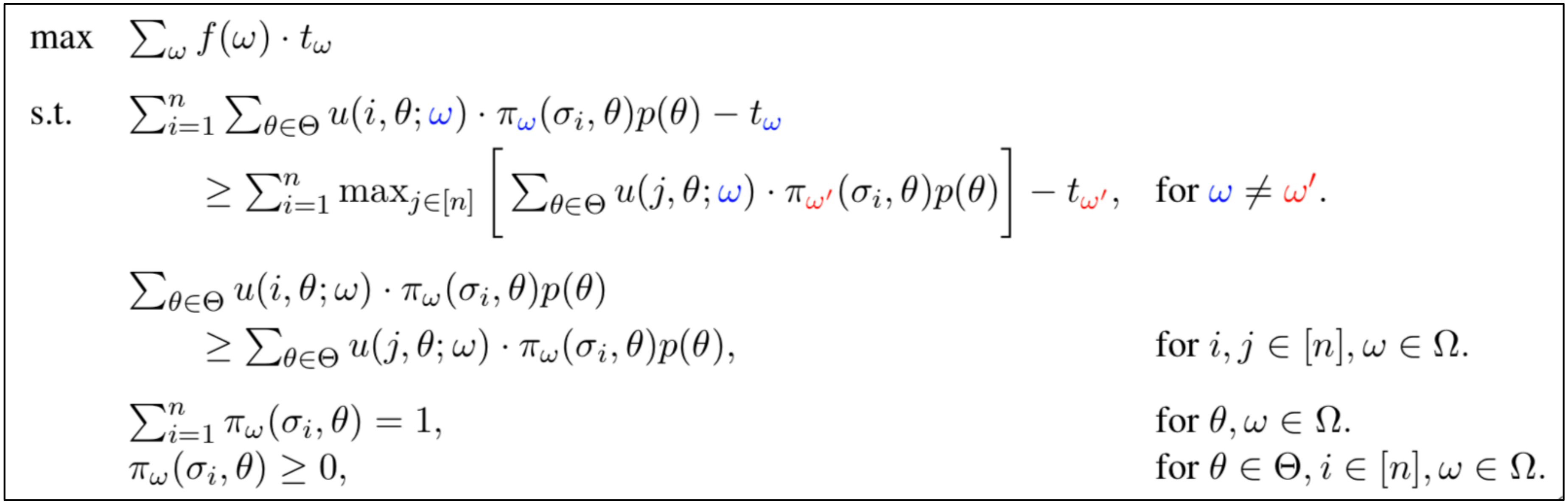
Theorem. Consulting mechanism is optimal with $\{t_\omega, \pi_\omega\}_\omega$ computed by the following program.

Sell Information: the Optimal Mechanism

Optimal $\{r_\omega, \pi_\omega\}_\omega$ can be computed by a convex program

- Variables: $\pi_\omega(\sigma_i, \theta)$ = prob of sending σ_i conditioned on θ for ω
- Variable t_ω is the payment from ω



$$
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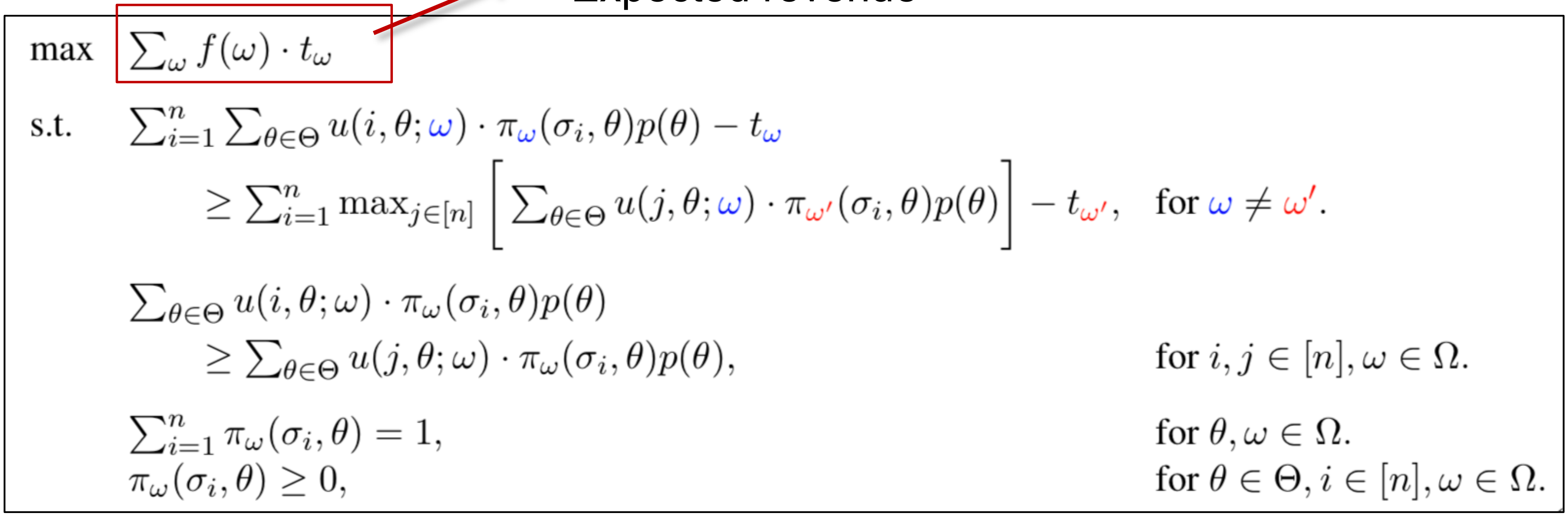
Expected revenue



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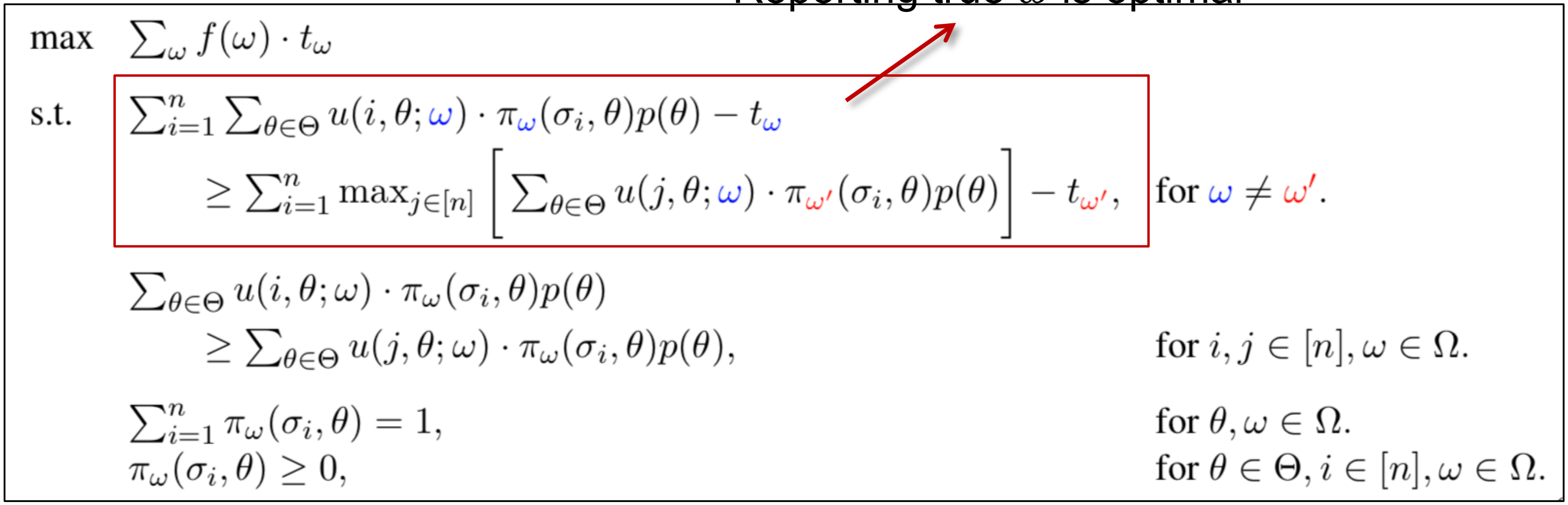
Reporting true ω is optimal



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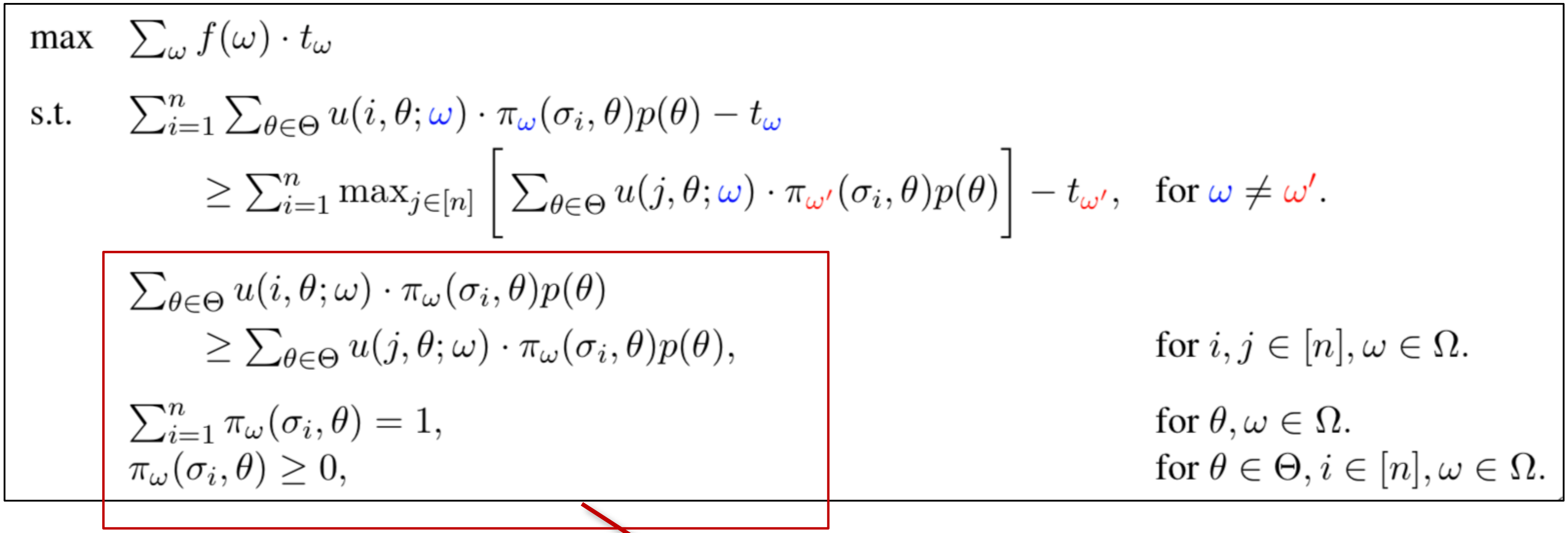

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Similar to constraints in persuasion

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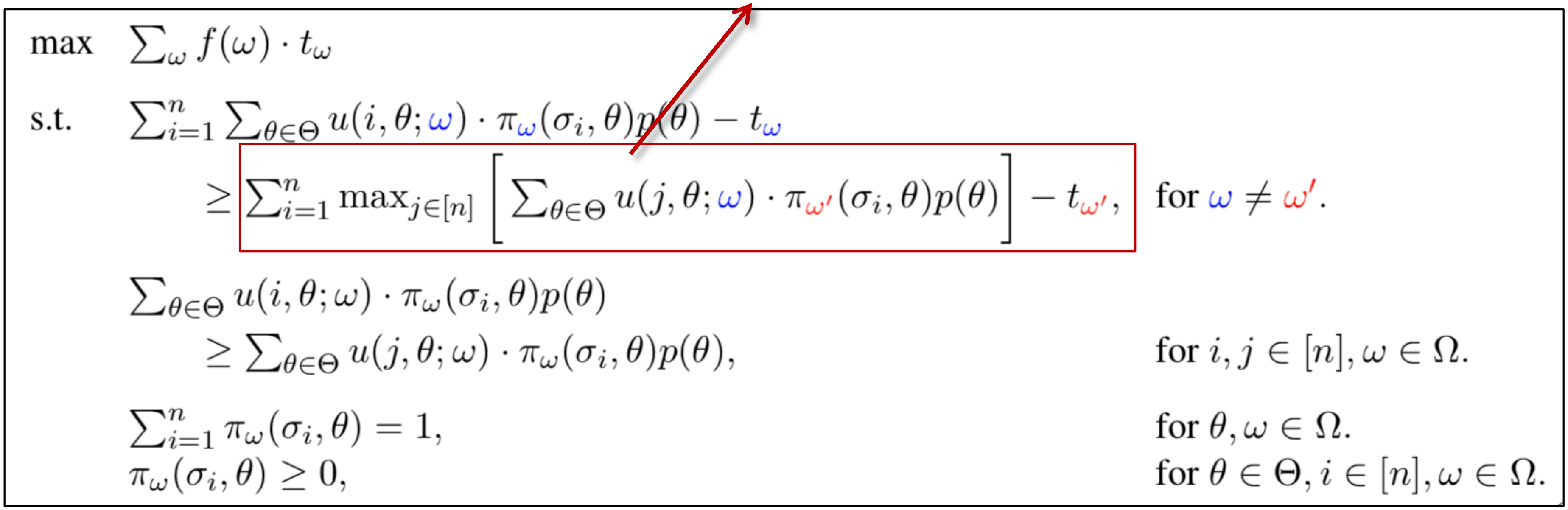
➢ A convex fnc of variables

➢ Can be converted to an LP



$$
\begin{array}{lll}
\max & \sum_\omega f(\omega) \cdot t_\omega & \\
\text{s.t.} & \sum_{i=1}^{n} \sum_{\theta \in \Theta} u(i, \theta; \omega) \cdot \pi_\omega(\sigma_i, \theta) p(\theta) - t_\omega & \\
& \quad \geq \boxed{\sum_{i=1}^{n} \max_{j \in [n]} \left[\sum_{\theta \in \Theta} u(j, \theta; \omega) \cdot \pi_{\omega'}(\sigma_i, \theta) p(\theta) \right] - t_{\omega'},} & \text{for } \omega \neq \omega'. \\
& \sum_{\theta \in \Theta} u(i, \theta; \omega) \cdot \pi_\omega(\sigma_i, \theta) p(\theta) & \\
& \quad \geq \sum_{\theta \in \Theta} u(j, \theta; \omega) \cdot \pi_\omega(\sigma_i, \theta) p(\theta), & \text{for } i, j \in [n], \omega \in \Omega. \\
& \sum_{i=1}^{n} \pi_\omega(\sigma_i, \theta) = 1, & \text{for } \theta, \omega \in \Omega. \\
& \pi_\omega(\sigma_i, \theta) \geq 0, & \text{for } \theta \in \Theta, i \in [n], \omega \in \Omega.
\end{array}
$$

Outline

- Bayesian Persuasion and Information Selling
- Sell to a Single Decision Maker
- Sell to Multiple Decision Makers

Challenges

- For single decision maker, more information always helps
  - Recall in persuasion, receiver always benefits from signaling scheme
- A fundamental challenge for selling to multiple buyers is that information does not necessarily help them

Example: More Information Hurts Buyers

- Insurance industry: *insurance company* and *customer*
  - Both are potential information buyers
- Two types of customers: Healthy and Unhealthy
  - Publicly know, Pr(Healthy) = 0.9
- Seller is an information holder, who knows whether any customer is healthy or not

Insurance company

customer

	Sell	Not Sell
Buy	(-10, 10)	(-0, 0)
Not Buy	(0 , 0)	(0 , 0)

Healthy customer

Insurance company

	Sell	Not Sell
Buy	(-10, -50)	(-110, 0)
Not Buy	(-111 , 0)	(-111 , 0)

Unhealthy customer

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Insurance company

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Healthy customer, prob = 0.9

Insurance company

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Buy	(-10, -50)	(-110, 0)
Not Buy	(-111 , 0)	(-111 , 0)

Unhealthy customer

Q: What happens without seller’s information ?

- Customer and insurance company will look at expectation
  - Dominant strategy equilibrium is (Buy, Sell)

	Sell	Not Sell
Buy	$\boxed{(-10, 4)}$	(-11 , 0)
Not Buy	(-11.1, 0)	(-11.1, 0)

Example: More Information Hurts Buyers

Insurance company

customer

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Not Buy	(0 , 0)	(0 , 0)

Healthy customer, prob = 0.9

Insurance company

	Sell	Not Sell
Buy	(-10, -50)	(-110, 0)
Not Buy	(-111 , 0)	(-111 , 0)

Unhealthy customer

Q: What if seller tells (only) customer her health status ?

E.g., customer wants to buy info from seller to decide whether he should buyer insurance or not

Example: More Information Hurts Buyers

Insurance company

customer

	Sell	Not Sell
Buy	(-10, 10)	(-0, 0)
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Unhealthy customer

Q: What if seller tells (only) customer her health status ?

- If Healthy, customer will not buy
- If Unhealthy, customer will buy
- Customer’s reaction reveals his healthy status

Example: More Information Hurts Buyers

Insurance company

customer

	Sell	Not Sell
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Unhealthy customer

Q: What if seller tells (only) customer her health status ?

- If Healthy, customer will not buy → utility (0,0) for both
- If Unhealthy, customer will buy → Will not sell, utility (-110,0)
- Customer's reaction reveals his healthy status

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- If Healthy, customer will not buy → utility (0,0) for both
- If Unhealthy, customer will buy → Will not sell, utility (-110,0)
- Customer's reaction reveals his healthy status
- In expectation (-11, 0)

Recall previously (-10,4)

Thank You

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