

# CS650 I: Topics in Learning and Game Theory (Fall 2019)

## Inherent Trade-Offs in Algorithmic Fairness

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Instructor: Haifeng Xu

# COMPAS: A Risk Prediction Tool to Criminal Justice

- Correctional Offender Management Profiling for Alternative Sanctions (COMPAS)
  - Used by states of New York, Wisconsin, Cali, Florida, etc.
  - A software that assesses likelihood of a defendant of reoffending
- Still many issues
  - Not interpretable
  - Low accuracy
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  - Bias/unfairness (this lecture)

# COMPAS: A Risk Prediction Tool to Criminal Justice

➤ In a ProPublica investigation of the algorithm...

“...blacks are almost twice as likely as whites to be labeled a higher risk but not actually re-offend” -- **unequal false positive rate**

“... whites are much more likely than blacks to be labeled lower-risk but go on to commit other crimes” -- **unequal false negative rate**

Algorithms seem unfair!!

# Other Examples

## ➤ Advertising and commercial contents

Search Engines

April 2, 2013

Volume 11, issue 3



### **Discrimination in Online Ad Delivery**

**Google ads, black names and white names, racial discrimination, and click advertising**

Latanya Sweeney

Searching names that are likely assigned to black babies generates more ads suggestive of an arrest

# Other Examples

- Advertising and commercial contents
  - If a male and female user are equally interested in a product, will they be equally likely to be shown an ad for it?
  - Will women in aggregate be shown ads for lower-paying jobs?
- Medical testing and diagnosis
  - Will treatment be applied uniformly across different groups of patients?
- Hiring or admission
  - Will students or job candidates from different groups be admitted with equal probability?
- ...

# Why Algorithms May Be “Unfair”?

- Algorithms may encode pre-existing bias
  - E.g., British Nationality act program, designed to automate evaluation of new UK citizens
  - It accurately reflects tenets of the law “a man is the father of only his legitimate children, whereas a woman is the mother of all her children, legitimate or not”

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- Algorithms may create bias when serving its own objective
  - E.g., search engines try to show your favorite contents but not the most fair contents
- Input data are biased
  - E.g., ML may classify based on sensitive features in biased data
  - Can we simply remove these sensitive features during training?
- Biased algorithm may get biased feedback and further strengthen the issue

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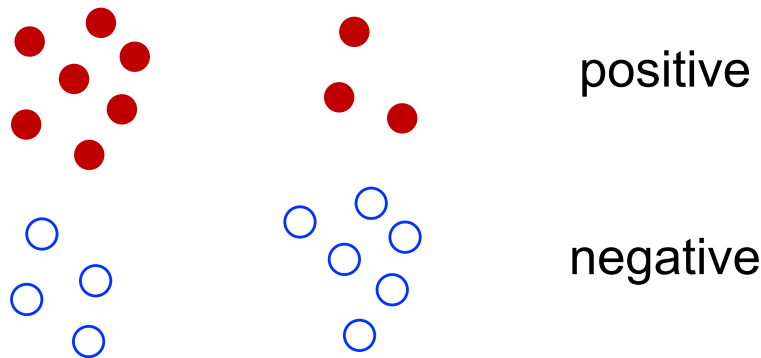
This lecture: there is another reason – some basic definitions of fairness are intrinsically not compatible

# The Problem of Predicting Risk Scores

- In many applications, we classify whether people possess some property by predicting a score based on their features
  - Criminal justice
  - Loan lending
  - University admission
- Next: an abstract model to capture this process

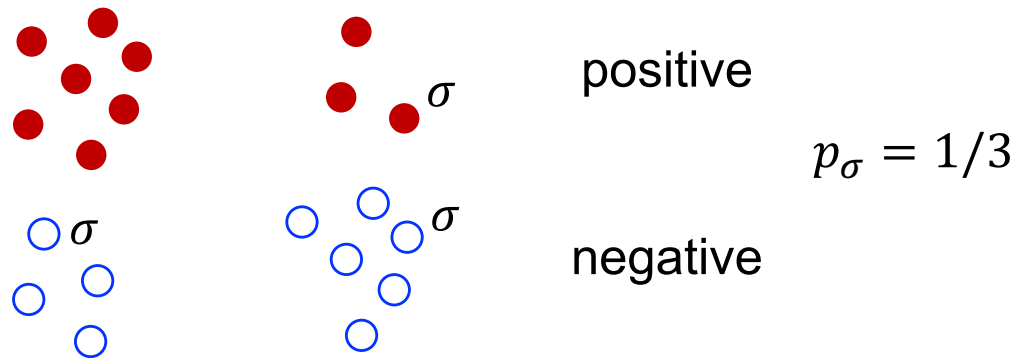
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- There is a collection of people, each of whom is either a positive or negative instance
  - Positive/negative describe the true label of each individual



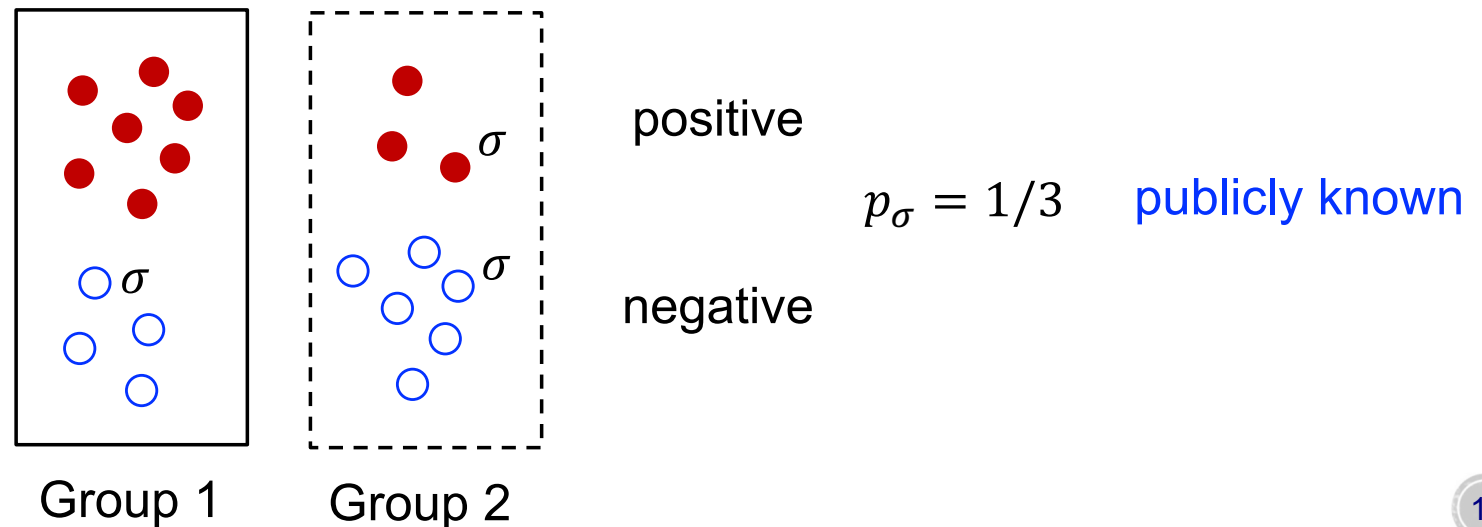
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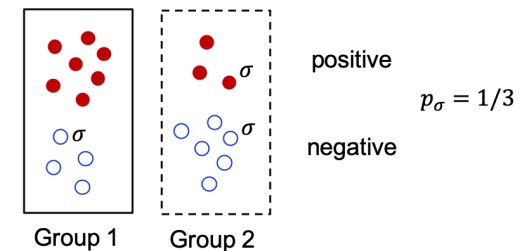
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- Each person belongs to one of two groups



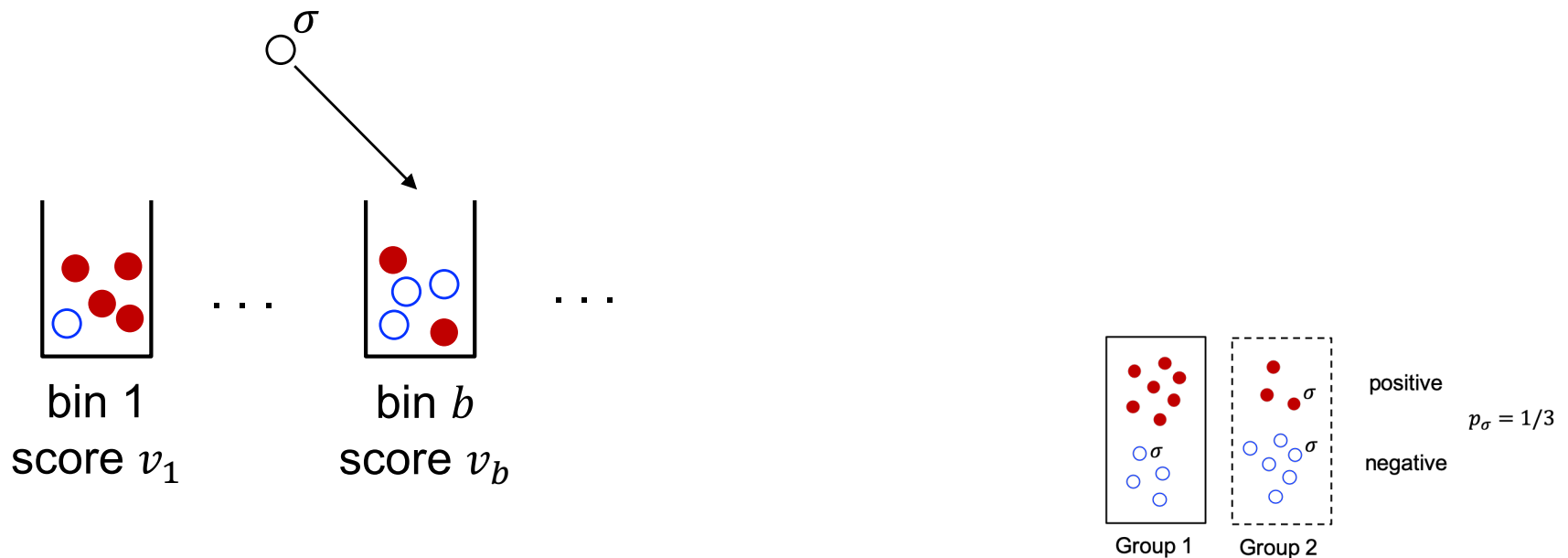
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- Task: assign risk score to each individual
- Objective: accuracy (of course) and “fair”
  - Naturally, the score should only depend on  $\sigma$ , not individual's group



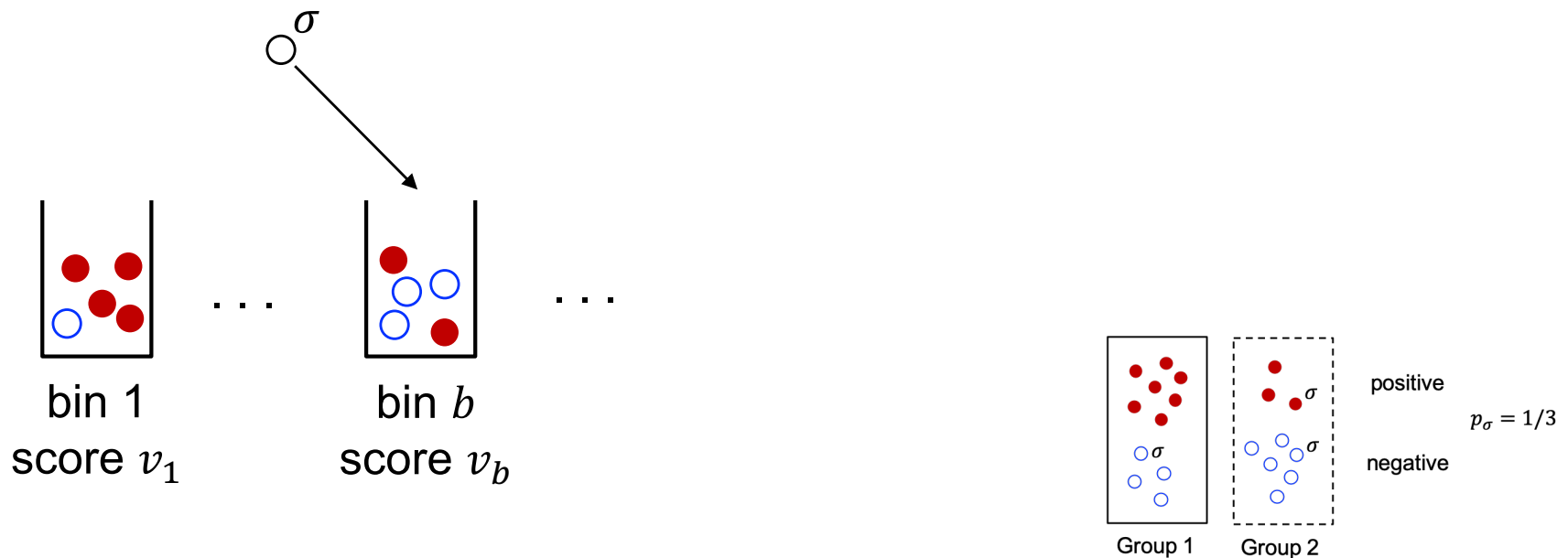
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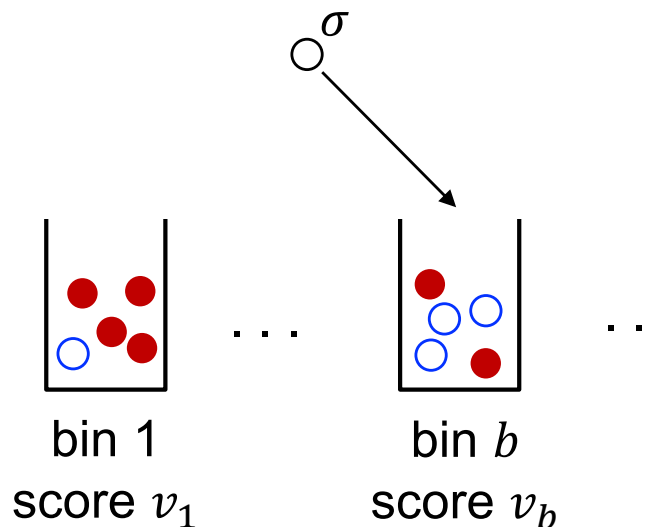
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  - Example 1: assign all  $\sigma$  to the same bin; give that bin score  $p_\sigma$
  - Example 2: assign all people to one bin; give score 1

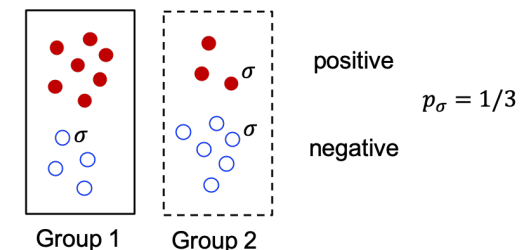


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Note: may have very bad accuracy but good fairness, as they are different



# Well...What Does “Fair” Really Mean?

- A very subjective perception
- Yet, for algorithm design, need a concrete and objective definition
- > 20 different definitions of fairness so far
  - See a survey paper “Fairness Definitions Explained”
- This raises many questions
  - Are they all reasonable? Can we satisfy all of them?
  - Which one/subset of them we should use when designing algorithms?
  - Do I have to sacrifice accuracy to achieve fairness?

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Some basic definitions of fairness are already not compatible, regardless how much accuracy you are willing to sacrifice

# Fairness Def I: Calibration

**Definition [Calibration within groups].** For each bin  $b$ , let

- $N_{t,b}$  = # of people assigned to  $b$  from group  $t$
- $n_{t,b}$  = # of **positive** people assigned to  $b$  from group  $t$

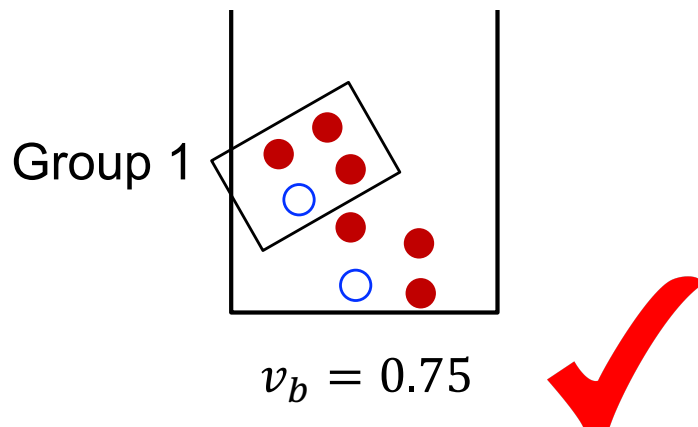
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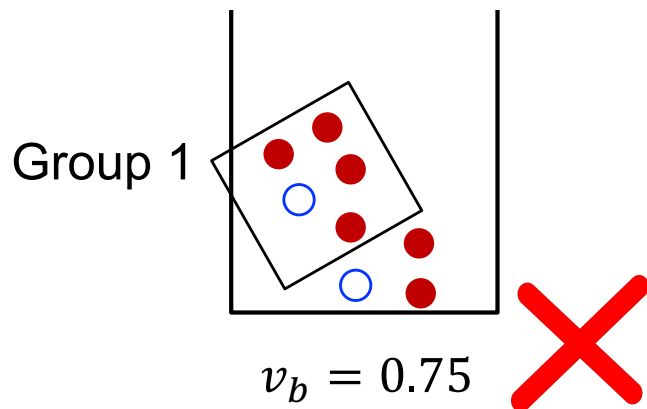


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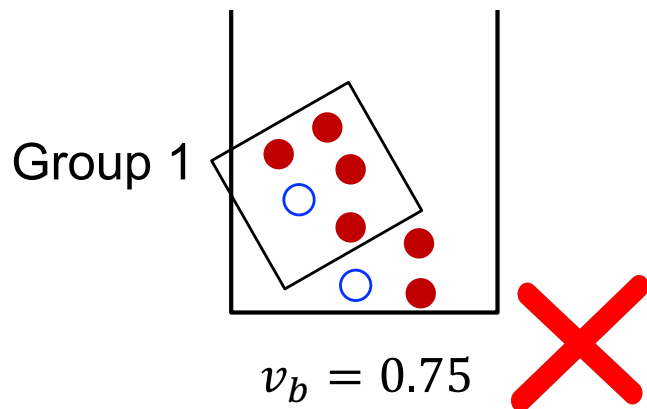


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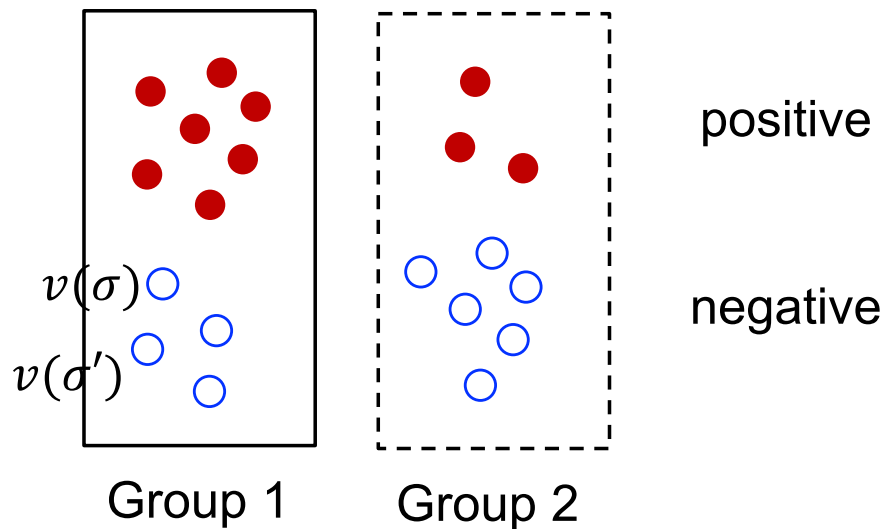
We should have  $n_{t,b} = v_b \cdot N_{t,b}$  for each  $t, b$



In practice, we do not know who are positive so cannot check the condition, but the definition still applies

# Fairness Def 2: Balance of Negative Class

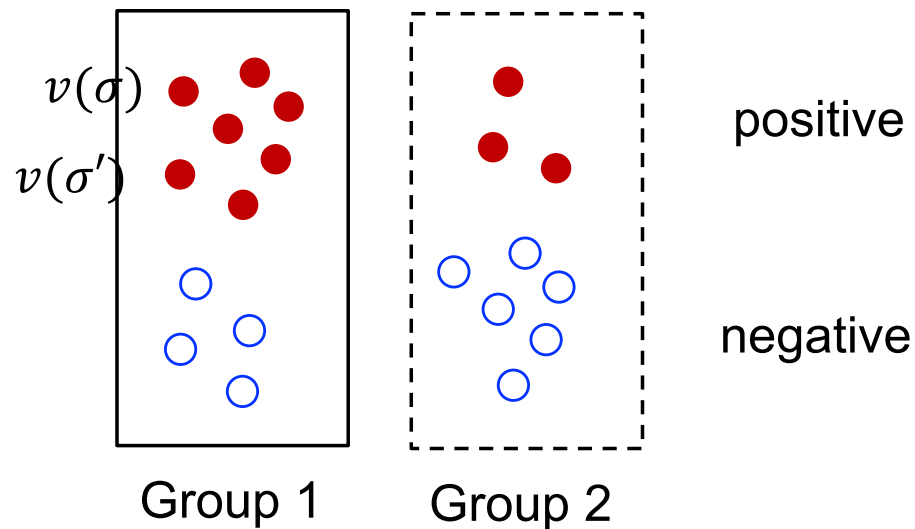
**Definition [Balance of Negative Class].** Average scores assigned to people of group 1 who are negative should be the same as average scores assigned to people of group 2 who are negative.



$$E[v(\sigma) | \sigma \text{ negative and in group 1}] \\ = E[v(\sigma) | \sigma \text{ negative and in group 2}]$$

# Fairness Def 3: Balance of Positive Class

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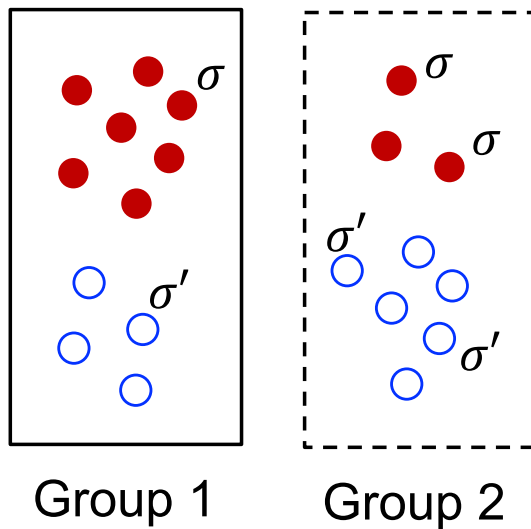
$$E[v(\sigma) | \sigma \text{ positive and in group 1}] \\ = E[v(\sigma) | \sigma \text{ positive and in group 2}]$$

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Yes: Example 1

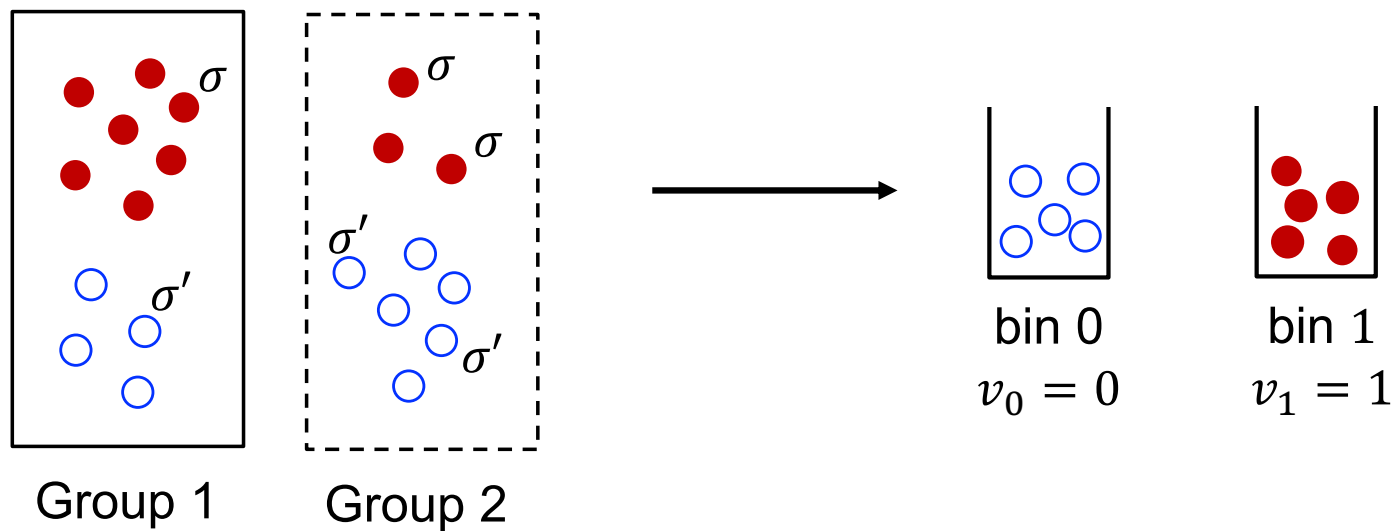
➤  $p_\sigma = 1$  or 0 for all  $\sigma$



# Is It Possible to Achieve All Three?

Yes: Example 1

- $p_\sigma = 1$  or 0 for all  $\sigma$
- Two bins with  $v_0 = 0$  and  $v_1 = 1$ ; assign all  $\sigma$  with  $p_\sigma = 0$  to bin 0 and all  $\sigma$  with  $p_\sigma = 1$  to bin 1

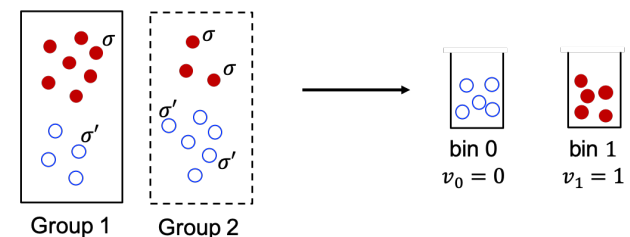


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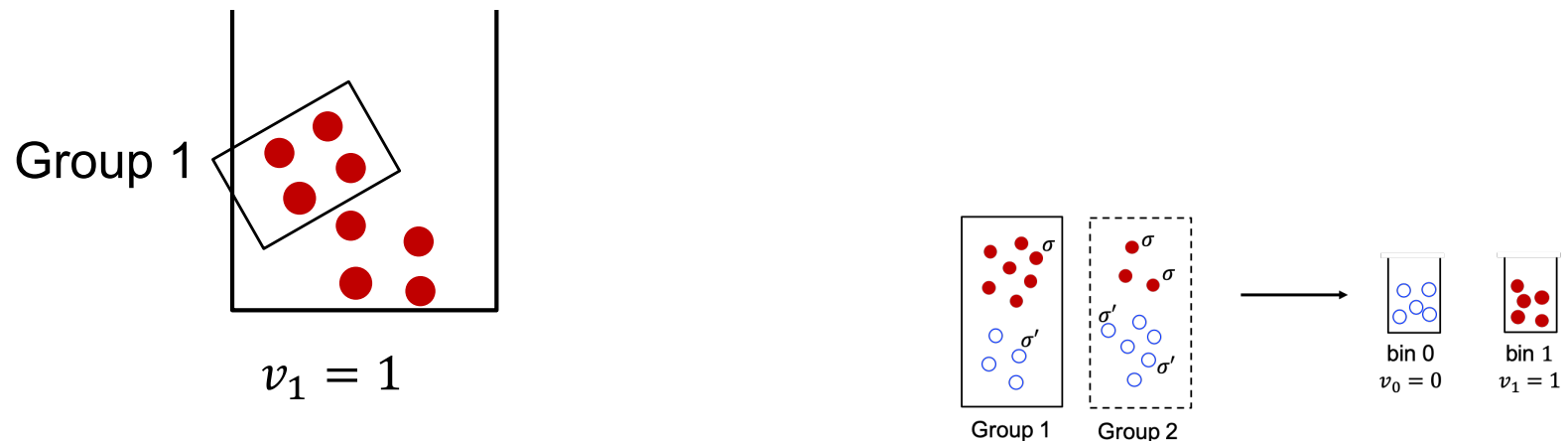
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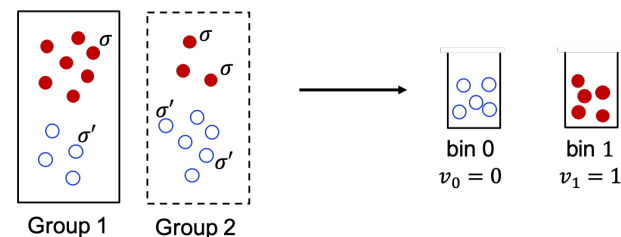
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- Calibration: yes, all the ratio is 1 or 0 for each group
- Balance of positive class: yes, both groups have average score 1
- Balance of negative class: yes, both groups have average score 0



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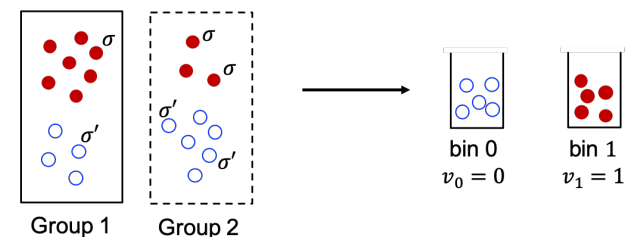
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Caveats

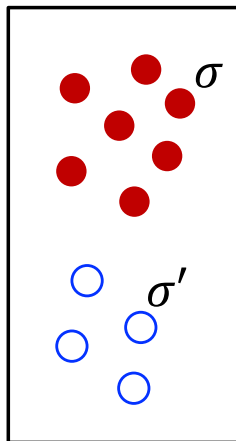
- But, this is **not really a realistic setting**...
- $p_\sigma = 0$  or 1 means we know for sure each individual's label



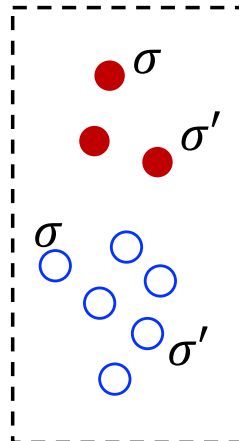
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Yes: Example 2

➤ Average  $p_\sigma$  (over  $\sigma$ 's) is the same among two groups



Group 1



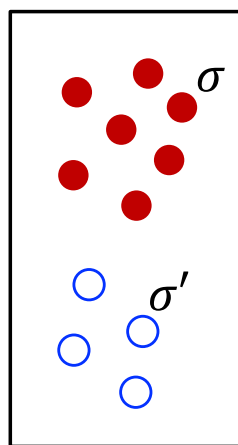
Group 2

$$E[p_\sigma | \sigma \in \text{Group 1}] = E[p_\sigma | \sigma \in \text{Group 2}]$$

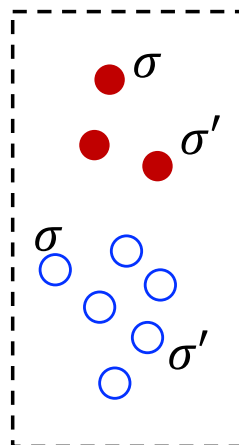
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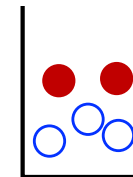
- Average  $p_\sigma$  (over  $\sigma$ 's) is the same among two groups
- One bin, with  $v$  equal the above average  $p_\sigma$



Group 1



Group 2



$$v = E[p_\sigma | \sigma \in \text{Group 1}]$$

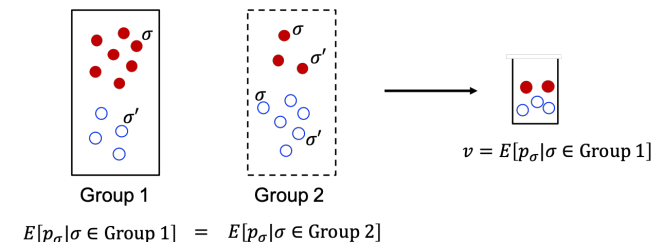
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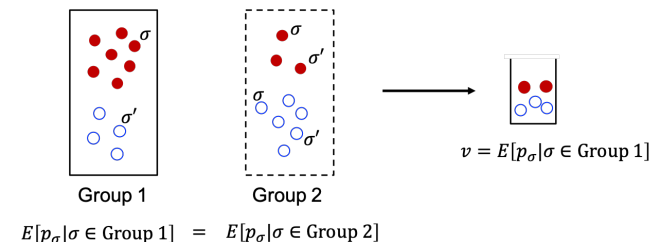
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**Claim:** This score assignment satisfies all 3 fairness defs.

- Calibration: yes, since  $v = \text{average } p_\sigma$  is exactly the probability of positive instances in both groups
- Balance of positive class: trivial, as scores are the same
- Balance of negative class: trivial as well



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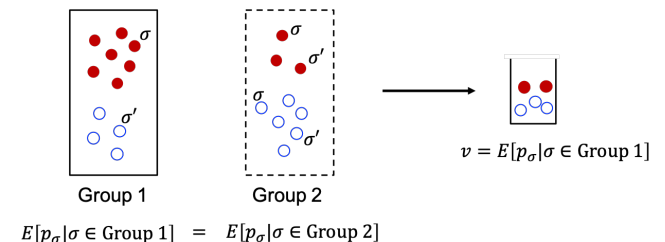
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Caveats

- But, this **score assignment is not useful and has low accuracy**
- There may exist a more accurate score assignment in this case that still satisfy three definitions
  - **Bad news: it is NP-hard to find**



# Inherent Trade-offs of Algorithmic Fairness

**Theorem:** For the problem of risk score assignment, if there is a risk assignment that satisfies all the three fairness definitions before, the problem must be one of the previous two example cases.

- The two (degenerated) examples are the only cases where you can possibly satisfy all three fairness definitions

# Proof Sketch

- Assume there is a score assignment satisfying all three defs
- Will derive contradictions, unless the instance is the previous degenerated settings

# Proof Sketch

## Notations

- $N_t$  = total number of people in group  $t$
- $n_t$  = total number of positive people in group  $t$

## Calibration condition implies

- Total score of all group- $t$  people in bin  $b$  (i.e.,  $v_b \cdot N_{t,b}$ ) equal expected number of positive group- $t$  people in bin  $b$  (i.e.,  $n_{t,b}$ )

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- Summing over all bins  $\rightarrow$  total score of all group- $t$  people equals expected number of positive group- $t$  people

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## Another way to calculate total scores

- $x$  = average score of a person in negative class
- $y$  = average score of a person in positive class

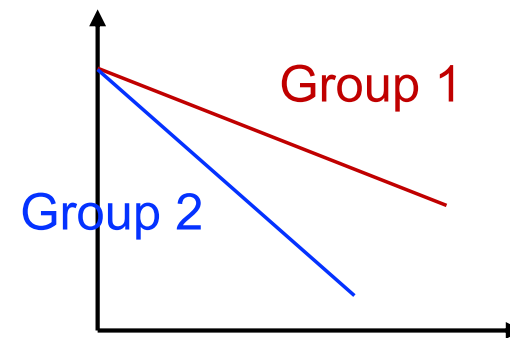
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- Re-arranging  $x = (1 - y) \frac{n_t}{N_t - n_t}$



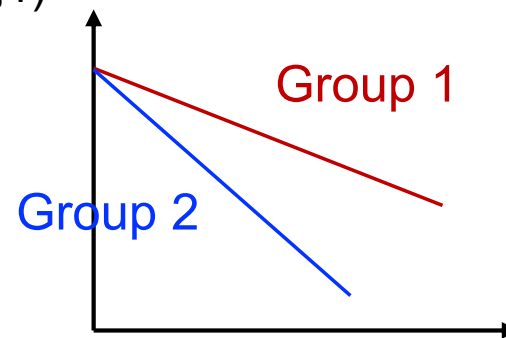
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- Re-arranging  $x = (1 - y) \frac{n_t}{N_t - n_t}$
- To make sure  $x, y$  are the same for both groups, the two lines must intersect
  - Unless slopes are the same, only intersect at  $(0,1)$



# Can Achieve Two Definitions

- “Equality of Opportunity in Supervised Learning [NeurIPS’16]”
  - Can achieve balance of positive and negative class, but no requirement for calibration
  - Objective: find most accurate prediction subject to fairness constraints
- “On Fairness and Calibration [NeurIPS’17]”
  - Can achieve calibration and any linear combination of balance of positive and negative class

# Similar Negative Results

“Fair prediction with disparate impact: A study of bias in recidivism prediction instruments”

“Algorithmic decision making and the cost of fairness”

➤ Show similar negative results, but for classification

Happy Thanksgiving